DIMENSIONAL ANALYSIS AND MODEL STUDIES

Dimensional analysis is a method of dimensions. It is a mathematical technique used for conducting model tests. It deals with the dimensions of the physical quantities involved in the phenomenon. Length L, Mass M and Time T are three fixed dimensions and are called as Fundamental dimensions or fundamental quantity.

Quantities which posses more than one fundamental dimension is called as Secondary quantity or derived quantity.

Ex: Velocity $= \frac{\text{Length}}{\text{Time}} = (LT^{-1})$

Quantities	Dimensions.
a) Length	L
b) Mass	M
c) Time	T
d) Area	L^2
e) Volume	L^3
f) Velocity	LT^{-1}
g) Momentum	MLT^{-1}
h) Torque	ML^2T^{-2}
i) Power	MLT^{-3}
j) Work, Energy	ML^2T^{-2}
k) Shear stress	$ML^{-1}T^{-2}$
l) Discharge	L^3T^{-1}
m) force	MLT^{-2}
n) Density	ML^{-3}
o) Dynamic Viscosity	$ML^{-1}T^{-1}$

(4) Determine the dimensions of the quantities given below (i) angular velocity (ii) angular acceleration (iii) Discharge (iv) Kinematic viscosity (v) force (vi) Specific weight.

Soln:

Angular velocity = $\frac{\text{angle covered in radians}}{\text{time}}$

$= \frac{1}{T} = T^{-1}$

angular acceleration = $\frac{\text{angle covered in radians}}{\text{time}^2}$

$= T^{-2}$

Discharge = Area × velocity

$= L^2 \times \frac{L}{T} = L^3 T^{-1}$

Kinematic viscosity (ν) = $\frac{\mu}{S}$ = $\frac{\text{Dynamic Viscosity}}{\text{Density}}$

$\tau = \mu \cdot \frac{du}{dy}$

$\frac{\text{Force}}{\text{Area}} = \mu \cdot \frac{\text{velocity}}{\text{length}}$

$\frac{L}{LT^{-1}} \times \frac{M \times LT^{-2}}{L^2} = \mu$

$\mu = ML^{-1}T^{-1}$

force = mass × acceleration due to gravity

$= M \times LT^{-2}$

$= MLT^{-2}$

Specific weight = $\frac{\text{Weight}}{\text{volume}}$

$= \frac{M \times LT^{-2}}{L^3} = \frac{MLT^{-2}}{L^3}$

$= ML^{-2}T^{-2}$

Dimensional Homogeneity.

If the dimension of each term in an equation on both sides are equal, then it is termed as "Dimensional homogeneity"

The powers of fundamental dimensions on both sides of the equation will be identical for a dimensionally homogeneous equation

Consider $V = \sqrt{2gH}$

$V = LT^{-1}$

$g = LT^{-2}$

$H = L$

$L.H.S = LT^{-1}$

$R.H.S = \sqrt{L^2T^{-2}} = \sqrt{\frac{L^2}{T^2}}$

$R.H.S = LT^{-1}$

METHODS OF DIMENSIONAL ANALYSIS.

1) RAYLEIGH'S METHOD :

This method is used for determining the expression for a variable which depends upon maximum three or four variables only.

Let 'x' be a variable which depends on x_1, x_2 and x_3 variables. Then according to Rayleigh x is a function of x_1, x_2 and x_3 which can be written as

$x = f[x_1, x_2, x_3]$

$\therefore x = K x_1^a \, x_2^b \, x_3^c$

where K is the constant

a & b are the arbitrarily powers

(2) The time period (t) of a pendulum depends upon the length (L) of the pendulum and acceleration due to gravity (g). Derive an expression for the time period.

Soln Time is a function of (i) L and (ii) g.

$$t = k \cdot L^a \cdot g^b$$

$$T = k \cdot L^a \cdot (LT^{-2})^b$$

equating the powers of M, L and T

for M; $0 = 0$

for L; $0 = a + b$
$a = -b$ $\therefore a = +1/2$

for T; $1 = -2b$
$b = -1/2$

$$t = k \cdot L^{1/2} \cdot g^{-1/2}$$

$$\boxed{t = k \cdot \sqrt{\frac{L}{g}}}$$

(3) The efficiency η of a fan depends on the density ρ, the dynamic viscosity μ of the fluid, the angular velocity ω, diameter D of the rotor and discharge Q. Express η in terms of dimensionless parameters.

Soln: η depends on (i) ρ, density (ii) μ, dynamic viscosity (iii) ω, angular velocity (iv) D, diameter (v) Q, discharge

Dimension

$$\rho = ML^{-3}$$

$$\mu = ML^{-1}T^{-1}$$

$$\omega = T^{-1}$$

$$D = L$$

$$Q = L^3T^{-1}$$

According to Rayleigh

$$\eta = k \cdot \rho^a \mu^b \cdot \omega^c \cdot D^d \cdot Q^e$$

$$M^0 L^0 T^0 = (ML^{-3})^a (ML^{-1}T^{-1})^b (T^{-1})^c \; L^d (L^3 T^{-1})^e$$

equating the powers

for M $\quad 0 = a + b \implies a = -b$

for L $\quad 0 = -3a - b + d + 3e \implies d = -2b - 3e$

for T $\quad 0 = -b - c - e \implies c = -b - e$.

$$\eta = K \cdot \rho^{-b} \cdot \mu^{b} \cdot \omega^{-b-e} \cdot D^{-2b-3e} \cdot Q^e$$

$$= K \cdot \rho^{-b} \cdot \omega^{-b} \mu^{b} \cdot D^{-2b} \cdot \omega^{-e} D^{-3e} Q^e$$

$$= k \left[\frac{\mu}{\rho \omega D^2}\right]^b \left[\frac{Q}{\omega D^3}\right]^e$$

$$= \phi \left[\left(\frac{\mu}{\rho \omega D^2}\right), \left(\frac{Q}{\omega D^3}\right)\right]$$

4) The resistance R, to the motion of a completely submerged body depends upon the length of the body L, velocity of flow V, mass density of fluid ρ and kinematic viscosity ν.

Find an expression for resistance.

Soln The resistance R depends on (i) length L, velocity V, mass density ρ, kinematic viscosity.

Dimension

$L = L$

$V = LT^{-1}$

$\rho = ML^{-3}$

$\nu = \cancel{ML^{-3}} L^2 T^{-1}$

According to Rayleigh.

$$R = k \cdot L^a \cdot (LT^{-1})^b \cdot (ML^{-3})^c \cdot (L^2 T^{-1})^d$$

$$MLT^{-2} = k \cdot L^a \cdot (LT^{-1})^b \cdot (ML^{-3})^c \cdot (L^2 T^{-1})^d.$$

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Equating the powers

for M : $1 = c$

for L : $1 = a + b - 3c + 2d$ $a = 1 - b + 3c - 2d$
$\Rightarrow a = 2 - d$

for T : $-2 = -b - d$ $b = 2 - d$.

$$R = k \cdot L^{(2-d)} \, V^{(2-d)} \cdot \rho \cdot \nu^{d}.$$

$$= k \cdot \frac{L^2}{L^d} \cdot \frac{V^2}{V^d} \cdot \rho \cdot \nu^d .$$

$$= k \cdot L^2 V^2 \rho \left[\frac{\nu}{LV} \right]^d$$

$$R = k \cdot L^2 V^2 \rho \; \phi \left[\frac{\nu}{LV} \right]$$

Q5> Find an expression for the drag force on smooth sphere of diameter D, moving with a uniform velocity V in a fluid of density ρ and dynamic viscosity μ.

Soln: Drag force depends on (i) Diameter D, (ii) Velocity V, (iii) Density ρ, (iv) Dynamic viscosity μ.

Dimensions

$D = L$

$V = LT^{-1}$

$\rho = ML^{-3}$

$\mu = ML^{-1}T^{-1}$

According to Rayleigh.

$$F = k \cdot D^a \cdot V^b \cdot \rho^c \cdot \mu^d .$$

$$MLT^{-2} = k \cdot L^a \cdot (LT^{-1})^b \cdot (ML^{-3})^c \cdot (ML^{-1}T^{-1})^d .$$

equating the powers

for M $1 = c + d \Rightarrow c = 1 - d$

L $1 = a + b - 3c - d \Rightarrow 1 = a + 2 - d - 3(1-d) - d$

T $-2 = -b - d \Rightarrow b = 2 - d$.

$1 - 2 + d + 3 - 3d + d = a$

$2 - d = a$

$$F = k \cdot D^{2-d} \cdot V^{2-d} \cdot \rho^{1-d} \cdot \mu^{d}$$

$$F = k \cdot D^2 V^2 \rho \left[\frac{\mu}{DV\rho}\right]^{d}$$

$$F = k \cdot D^2 V^2 \cdot \rho \; \phi\left[\frac{\mu}{DV\rho}\right]$$

BUCKINGHAM'S π THEOREM

"If there are n variables in a physical phenomenon and if these variables contain m fundamental dimension (M, L, T) then the variables are arranged into $(n-m)$ dimensionless terms. Each term is called π term.

Let x_1 be the dependent variables and $x_2, x_3 \ldots x_n$ are the independent variables on which x_1 depends. Then x_1 is a function of $x_2, x_3 \ldots x_n$.

$$x_1 = f(x_2, x_3 \ldots x_n)$$

$$\therefore f_1(x_1, x_2, x_3 \ldots x_n) = 0.$$

It contains n variables. If there are m fundamental dimension then it can be written in terms of number of dimensionless groups or π terms in which number of π terms is equal to $(n-m)$.

$$f(\pi_1, \pi_2 \ldots \pi_{n-m}) = 0.$$

Each of π term is dimensionless and contains $m+1$ variables, where m is the number of fundamental dimensions and is called the repeating variables.

Ex: If there are four variables. Each π term is written as

$$\pi_1 = x_2^{a_1} x_3^{b_1} \cdot x_4^{c_1} \cdot x_1$$

$$\pi_2 = x_2^{a_2} \cdot x_3^{b_2} \cdot x_4^{c_2} \cdot x_5$$

$$\vdots$$

$$\pi_{n-m} = x_2^{a_{n-m}} \cdot x_3^{b_{n-m}} \cdot x_4^{c_{n-m}} \cdot x_n.$$

Each equation is solved by the principle of dimensional homogeneity and values of a_1, b_1, c_1 etc are obtained and substituted and $\pi_1, \pi_2 \ldots \pi_{n-m}$ are obtained.

$$\pi_1 = \phi\left[\pi_2, \pi_3 \ldots \pi_{n-m}\right]$$
$$\pi_2 = \phi\left[\pi_1, \pi_3 \ldots \pi_{n-m}\right].$$

Method of Selecting Repeating Variables.

1) As far as possible, the dependent variable should not be selected as repeating variable.

2) The variables should be choosen such that one variable contains geometric property, the other contains flow property and third contains fluid property.

Geometric property.
(i) length (ii) d (iii) height.

flow property.
(i) velocity (ii) acceleration.

fluid property.
(i) μ, (ii) ρ (iii) w.

3) No repeating variables should have the same dimensions.

4) The repeating variables together must have the same number of fundamental dimensions.

(5) The resisting force R of a supersonic plane during flight can be considered as dependent upon the length of the aircraft l, velocity V, air viscosity μ, air density ρ and bulk modulus of air K. Express the functional relationship b/n these variables and the resisting force.

Soln: Resisting force depends upon

(1) length of the aircraft $= l = L$

(2) velocity $V = LT^{-1}$

(3) viscosity $\mu = ML^{-1}T^{-1}$

(4) density $\rho = ML^{-3}$

(5) Bulk Modulus $K = ML^{-1}T^{-2}$

$$\therefore f_1(R, L, V, \mu, \rho, K) = 0$$

No of variables $n = 6$

No of fundamental dimensions $m = 3$

No of dimensionless π terms $= n - m = 3$ π-terms

π_1, π_2 and π_3 terms are formed

$$f_1(\pi_1, \pi_2, \pi_3) = 0$$

Each π term $= m+1$ variables, where $m = 3$ and these are called repeating variables

Out of 6 variables, 3 has to be taken as repeating variables out of which one variable should have geometric property, second variable should have flow property and third one have fluid property

Each π term is written as

$$\pi_1 = l^{a_1} \cdot V^{b_1} \cdot \rho^{c_1} \cdot R$$

$$\pi_2 = l^{a_2} \cdot V^{b_2} \cdot \rho^{c_2} \cdot \mu$$

$$\pi_3 = l^{a_3} \cdot V^{b_3} \cdot \rho^{c_3} \cdot K$$

Each π term is solved by the principle of dimensional homogeneity.

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$\pi_1 = M^0L^0T^0 = L^{a_1} \cdot (LT^{-1})^{b_1} \cdot (ML^{-3})^{c_1} \, MLT^{-2}$

equating the powers

for $M \implies 0 = c_1 + 1 \implies c_1 = -1$

for $L \implies 0 = a_1 + b_1 - 3c_1 + 1 \quad a_1 = -2$

for $T \implies 0 = -b_1 - 2 \quad b_1 = -2$

substituting the values of a_1, b_1, c_1 in π_1

$$\pi_1 = l^{-2} \cdot V^{-2} \cdot \rho^{-1} \cdot R$$

$$\boxed{\pi_1 = \frac{R}{l^2 V^2 \rho}}$$

$\pi_2 = M^0L^0T^0 = L^{a_2} \cdot (LT^{-1})^{b_2} \cdot (ML^{-3})^{c_2} \cdot ML^{-1}T^{-1}$

equating the powers

for $M \implies 0 = +c_2 + 1 \implies c_2 = -1$

for $L \implies 0 = a_2 + b_2 - 3c_2 - 1 \quad a_2 = -1$

for $T \implies 0 = -b_2 - 1 \implies b_2 = -1$

substituting the values of a_2, b_2, c_2 in π_2.

$$\pi_2 = l^{-1} \cdot V^{-1} \cdot \rho^{-1} \cdot \mu$$

$$\boxed{\pi_2 = \frac{\mu}{l V \rho}}$$

$\pi_3 = M^0L^0T^0 = L^{a_3} (LT^{-1})^{b_3} \cdot (ML^{-3})^{c_3} \cdot ML^{-1}T^{-2}$.

equating the powers

for $M \implies 0 = c_3 + 1 \implies c_3 = -1$

for $L \implies 0 = a_3 + b_3 - 3c_3 - 1 \implies a_3 = 0$.

for $T \implies 0 = -b_3 - 2 \implies b_3 = -2$.

substituting the values of a_3, b_3, c_3 in π_3.

$$\pi_3 = l^0 \cdot V^{-2} \cdot \rho^{-1} \cdot K$$

$$\boxed{\pi_3 = \frac{K}{V^2 \rho}}$$

(ii)

$$f_1\left[\frac{R}{\rho l^2 v^2}, \frac{\mu}{l v \rho}, \frac{k}{v^2 \rho}\right] = 0$$

$$R = \rho l^2 v^2 \left[\frac{\mu}{l v \rho}, \frac{k}{v^2 \rho}\right]$$

Using Buckingham's π theorem, show that the discharge Q consumed by an oil ring is given by

$$Q = N d^3 \phi\left[\frac{\mu}{\rho N d^2}, \frac{\sigma}{\rho N^2 d^3}, \frac{w}{\rho N^2 d}\right]$$

where d is the internal diameter of the ring, N is the rotational speed, ρ is density, μ is viscosity, σ is surface tension and w is the specific weight of oil.

Soln

$$Q = f(d, N, \rho, \mu, \sigma, w)$$

No of variables = 7.

Dimensions
$Q = L^3 T^{-1}$
$d = L$
$N = T^{-1}$
$\rho = ML^{-3}$
$\mu = ML^{-1}T^{-1}$
$\sigma = MT^{-2}$

Total number of fundamental dimensions, $m = 3$

No of π terms $= n - m = 7 - 3 = 4$ π terms.

Choosing d, N, ρ as repeating variables

$$\pi_1 = d^{a_1} \cdot N^{b_1} \cdot \rho^{c_1} \cdot Q$$

$$\pi_2 = d^{a_2} \cdot N^{b_2} \cdot \rho^{c_2} \cdot \mu$$

$$\pi_3 = d^{a_3} \cdot N^{b_3} \cdot \rho^{c_3} \cdot \sigma$$

For π_1 term
equating powers on both sides

$$M^0 L^0 T^0 = L^{a_1} \cdot (T^{-1})^{b_1} (ML^{-3})^{c_1} L^3 T^{-1}$$

for M ; $0 = c_1$

L : $0 = a_1 - 3c_1 + 3$ $a_1 = -3$

T ; $0 = -b_1 - 1 \Rightarrow b_1 = -1$

$$\pi_1 = d^{-3} N^{-1} \rho^{0} Q$$

$$= \frac{Q}{d^3 N}$$

for π_2 term

$$\pi_2 = d^{a_2} \cdot N^{b_2} \cdot \rho^{c_2} \cdot \mu$$

$$M^0 L^0 T^0 = L^{a_2} \cdot (T^{-1})^{b_2} \cdot (ML^{-3})^{c_2} \cdot ML^{-1}T^{-1}$$

for M ; $0 = c_2 + 1$ $\quad c_2 = -1$

L ; $0 = a_2 - 3c_2 - 1$ $\quad a_2 = -2$

T ; $0 = -b_2 - 1$ $\quad b_2 = -1$

Substituting

$$\pi_2 = d^{-2} \cdot N^{-1} \cdot \rho^{-1} \cdot \mu$$

$$\pi_2 = \frac{\mu}{d^2 N \rho}$$

for π_3 term

$$\pi_3 = d^{a_3} \cdot N^{b_3} \cdot \rho^{c_3} \cdot \sigma$$

$$M^0 L^0 T^0 = L^{a_3} \cdot (T^{-1})^{b_3} \cdot (ML^{-3})^{c_3} \cdot MT^{-2}$$

for M ; $0 = c_3 + 1 \implies c_3 = -1$

L ; $0 = a_3 - 3c_3 \implies a_3 = -3$

T ; $0 = -b_3 - 2 \implies b_3 = -2$

$$\pi_3 = d^{-3} \cdot N^{-2} \cdot \rho^{-1} \sigma$$

$$\pi_3 = \frac{\sigma}{d^3 N^2 \rho}$$

for π_4 term

$$\pi_4 = d^{a_4} \cdot N^{b_4} \cdot \rho^{c_4} \cdot \omega$$

$$M^0 L^0 T^0 = L^{a_4} \cdot (T^{-1})^{b_4} \cdot (ML^{-3})^{c_4} \cdot ML^{2}T^{-2}$$

for M ; $0 = c_4 + 1$ $\quad c_4 = -1$

L ; $0 = a_4 - 3c_4 - 2$ $\quad a_4 = -1$

T ; $0 = -b_4 - 2$ $\quad b_4 = -2$

$$\pi_4 = d^{-1} \cdot N^{-2} \cdot \rho^{-1} \cdot w$$

$$\pi_4 = \frac{w}{dN^2\rho}$$

$$f\left[\frac{Q}{d^3N}, \frac{\mu}{d^2N\rho}, \frac{\sigma}{d^3N^2\rho}, \frac{w}{dN^2\rho}\right]$$

$$\frac{Q}{d^3N} = f_1\left[\frac{\mu}{d^2N\rho}, \frac{\sigma}{d^3N^2\rho}, \frac{w}{dN^2\rho}\right]$$

$$Q = Nd^3 f_1\left[\frac{\mu}{d^2N\rho}, \frac{\sigma}{d^3N^2 d}, \frac{w}{dN^2\rho}\right]$$

Using Buckingham's π theorem, show that the discharge Q consumed by an oil ring is given by.

$$Q = Nd^3\phi\left[\frac{\mu}{\rho N d^2}, \frac{\sigma}{\rho N^2 d^3}, \frac{w}{\rho N^2 d}\right]$$

where d is the internal diameter of the ring, N is the rotational speed, ρ is density, μ is viscosity, σ is surface tension and w is the specific weight of oil.

$$Q = f($$

The frictional torque T of a disc of diameter D rotating at a speed N in a fluid of viscosity μ and density ρ in a turbulent flow is given by

$$T = D^5 N^2 \rho \,\phi\left[\frac{\mu}{D^2 N \rho}\right]$$

Prove this by the method of dimensions.

$$f_1 = [T, D, N, \mu, \rho]$$

No of variables $n = 5$

No of fundamental dimensions $m = 3$

No of π terms $= n - m = 5 - 3$ π terms.

π_1 and π_2 terms.

$$\pi_1 = D^{a_1} N^{b_1} \rho^{c_1} \cdot T$$

$$M^0 L^0 T^0 = L^{a_1} \cdot (T^{-1})^{b_1} \cdot (ML^{-3})^{c_1} \cdot ML^2 T^{-2}$$

equating the powers

for M ; $0 = c_1 + 1$ $\quad c_1 = -1$

for L ; $0 = a_1 - 3c_1 + 2$ $\quad a_1 = -5$

for T ; $0 = -b_1 - 2$ $\quad b_1 = -2$.

$$\pi_1 = D^{-5} \cdot N^{-2} \cdot \rho^{-1} \cdot T$$

$$\pi_1 = \frac{T}{D^5 N^2 \rho}$$

$$\pi_2 = D^{a_2} \cdot N^{b_2} \cdot \rho^{c_2} \cdot \mu$$

$$M^0 L^0 T^0 = L^{a_2} \cdot (T^{-1})^{b_2} \cdot (ML^{-3})^{c_2} \cdot ML^{-1} T^{-1}$$

equating the powers.

for M : $0 = c_2 + 1 \Rightarrow c_2 = -1$

L ; $0 = a_2 - 3c_2 - 1 \Rightarrow a_2 = -2$

T ; $0 = -b_2 - 1$ $\quad b_2 = -1$

$$\pi_2 = D^{-2} \cdot N^{-1} \cdot \rho^{-1} \cdot \mu$$

$$\pi_2 = \frac{\mu}{D^2 N \rho}$$

$$f\left[\frac{T}{D^2N^2S},\ \frac{\mu}{D^2NS}\right]=0$$

$$T=\underline{\underline{D^2N^2S}}\,\phi\left[\frac{\mu}{D^2NS}\right]$$

Model Studies

For predicting the performance of the hydraulic structures or hydraulic machines, before constructing or manufacturing, models of the structures or machines are made and tests are performed on them to obtain the desired information.

Model : It is the small scale replica of the actual structure or machine.

Prototype : The actual structure or machine is called Prototype

The study of models of actual machines is called Model analysis

Advantages of Dimensional Analysis and Model Studies

1) The performance of the hydraulic structure or hydraulic machine can be easily predicted in advance from its model

2) With the help of dimensional analysis, a relationship between the variables influencing the problem can be developed

3) Alternative designs can be predicted with the help of model testing.

Similitude

It is defined as the similarity between the model and its prototype in every respect, which means that the model and prototype have similar properties or model and prototype are completely similar

The three types of similarity are

1) Geometric Similarity
2) Kinematic Similarity.
3) Dynamic Similarity

Geometric Similarity : This similarity is said to exist between the model and the prototype if the ratio of all corresponding linear dimension in the model and prototype are equal.

$$\frac{L_p}{L_m} = \frac{b_p}{b_m} = \frac{D_p}{D_m} = L_r,$$ then there exists a geometric similarity.

where L_p = Length of prototype

b_p = Breadth of prototype

D_p = Diameter of prototype.

L_r = Scale ratio.

L_m, b_m, D_m = corresponding values of model.

For area's ratio

$$\frac{A_p}{A_m} = \frac{L_p \times b_p}{L_m \times b_m} = L_r \times L_r = L_r^2.$$

for volume's ratio

$$\frac{V_p}{V_m} = \frac{A_p \times L_p}{A_m \times L_m} = L_r^2 \times L_r^1 = L_r^3$$

Kinematic Similarity : This similarity is said to exist between the model and the prototype if the ratio of velocity and acceleration at the corresponding points in the model and prototype are same.

For Kinematic Similarity.

$$\frac{V_{p_1}}{V_{m_1}} = \frac{V_{p_2}}{V_{m_2}} = V_r = \text{velocity ratio}.$$

$$\frac{a_{p_1}}{a_{m_1}} = \frac{a_{p_2}}{a_{m_2}} = a_r = \text{acceleration ratio}.$$

wher V_{p_1} = velocity of fluid at point 1 in prototype.

V_{p_2} = " " " 2 in prototype

a_{p_1} = acceleration of fluid at point 1 in prototype.

a_{p_2} = " " at point 2 in protype.

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V_{m_1}, V_{m_2} and V_{m_1}, V_{m_2} = corresponding values of the model

Dynamic Similarity : It means the similarity of forces between the model and prototype if the ratios of the corresponding forces of the prototype and the model should be same.

For dynamic similarity

$$\frac{(F_i)_p}{(F_i)_m} = \frac{(F_v)_p}{(F_v)_m} = \frac{(F_g)_p}{(F_g)_m} = F_r = \text{force ratio}$$

$(F_i)_m$ = inertia force at a point in the model
$(F_v)_m$ = Viscous force at a point in the model
$(F_g)_m$ = Gravity force at a point in the model
$(F_i)_p, (F_v)_p, (F_g)_m$ = corresponding values in the prototype

DIMENSIONLESS NUMBER

These are the numbers obtained by dividing the inertia force by viscous force or gravity force or pressure force or surface tension force or elastic force. These are non dimensional parameters

Reynold's number : It is the ratio of inertia force of the flowing fluid and the viscous force of the fluid

Inertia force (F_i) = mass × acceleration of flowing fluid

$$= \rho \times \frac{\text{volume}}{\text{time}} \times \text{velocity}$$

$$= \rho \times A \times V \times V$$

$$= \rho A V^2$$

Viscous force (F_v) = Shear stress × Area

$$= \tau \times A$$

$$= \mu \frac{du}{dy} \times A = \mu \cdot \frac{V}{L} \times A$$

$$Re = \frac{\rho A V^2}{\mu \frac{V}{L} \times A} = \frac{\rho V L}{\mu}$$

In case of pipe flow

$$Re = \frac{\rho V d}{\mu}$$

Froude's number : It is the square root of the ratio of inertia force of a flowing fluid to the gravity force

$$F_e = \sqrt{F_i / F_g}$$

$$F_i = \rho A V^2$$

F_g = mass × accln to due to gravity

$= \rho \times \text{volume} \times g$

$= \rho \times A \times L \times g$

$$F_e = \sqrt{\frac{\rho A V^2}{\rho A L g}}$$

$$= \frac{V}{\sqrt{Lg}}.$$

MODEL LAW

Reynold's Model law :

This model is based on Reynold's number. It includes (i) pipe flow

(ii) Resistance experienced by submarines, airplanes, fully immersed bodies.

"The Reynold number for the model must be equal to the Reynold number for the prototype."

According to Reynold's law

$$\frac{\rho_m V_m L_m}{\mu_m} = \frac{\rho_p V_p L_p}{\mu_p}.$$

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11 A pipe of diameter 1.5m is required to transport an oil of G 0.90 and viscosity 3×10^{-2} poise at the rate of 3000 lps. Tests were conducted on a 15 cm diameter pipe using water at 20°C. Find the velocity and rate of flow in the model. Viscosity of water at 20°c = 0.01 poise

Soln: Prototype.

$D_p = 1.5m$.

$G = 0.9$

$\mu = 3\times10^{-2}$ poise

$Q = 3000$ lps.

$0.9 = \frac{\rho_p}{\rho_w}$

0.9×1000

Model.

$D_m = 15\times10^{-2} m$.

$V_m = ?$

$Q_m = ?$

$\mu_m = 0.01$ poise

$\rho_m = 1000 kg/m^3$

According to Reynold's ~~model~~ law

$$\frac{\rho_m V_m D_m}{\mu_m} = \frac{\rho_p V_p D_p}{\mu_p}.$$

$$\frac{1000\times V_m\times15\times10^{-2}}{0.01} = \frac{(0.9\times1000)\times V_p\times1.5}{3\times10^{-2}}$$

$$\frac{V_p}{V_m} = 1/3 = V_r.$$

$Q_p = A_p \times V_p$

$$\frac{3000\times10^{-3}}{\frac{\pi(1.5)^2}{4}} = V_p$$

$V_p = 1.697 m/s$.

$\therefore V_m = 3\times1.697$

$= 5.091 m/s$.

$V_m = \underline{3\times V_p}$

$Q_m = V_m \times A_m$

$= 5.091 \times \frac{\pi(15\times10^{-2})^2}{4}$

$= \underline{0.0899} m^3/s$.

The ratio of lengths of a sub marine and its model is 30 : 1. The speed of submarine (prototype) is 10 m/s. The model is to be tested in a wind tunnel. Find the speed of air in wind tunnel. Also determine the ratio of the drag (resistance) b/n the model and its prototype. Take the value of kinematic viscosities for sea water and air as 0.012 stokes and 0.016 stokes respectively. The density for sea water and air is given as 1030 kg/m³ and 1.24 kg/m³ respectively.

Prototype.

$V_p = 10$ m/s

$\nu_p = 0.012$ stokes.

$\rho_p = 1030$ kg/m³.

Model.

$V = ?$

$\nu = 0.016$

$\rho_m = 1.24$ kg/m³

$$\frac{F_p}{F_m} = ?$$

$$\frac{L_p}{L_m} = \frac{30}{1}$$

According to Reynold's Model law

$$\frac{\rho_p V_p L_p}{\mu_p} = \frac{\rho_m V_m L_m}{\mu_m}$$

$$\frac{10 \times 30}{0.012} = \frac{V_m \times 1}{0.016}$$

$$V_m = 400 \text{ m/s}$$

Resistance force (F) = mass × acceleration

$$= \rho \times L^3 \times \frac{V}{t}$$

$$= \rho L^2 V^2$$

$$\frac{F_p}{F_m} = \frac{\rho_p L_p^2 V_p^2}{\rho_m L_m^2 V_m^2} = \frac{10^2 \times 30^2 \times 1030}{1.24 \times 400^2 \times 1^2}$$

$$\frac{F_p}{F_m} = 467.23$$

A ship 300 m long moves in sea water, whose density is 1030 kg/m³. A 1:100 model of this ship is to be tested in a wind tunnel. The velocity of air in the wind tunnel around the model is 30 m/s and the resistance of the model is 60 N. Determine the velocity of ship in sea water and also the resistance of the ship in sea water. The density of air is given as 1.24 kg/m³. Take the kinematic viscosity of sea water and air as 0.012 stokes and 0.018 stokes respectively.

Prototype

ship in sea water

$\rho_p = 1030 \, kg/m^3$

$V_p = ?$

$F_p = ?$

$\nu_p = \frac{\mu_p}{\rho_p} = 0.012$ stokes

$L_p = 300m$

Model

wind tunnel.

$V_m = 30 \, m/s$

$F_m = 60N$

$\rho_m = 1.24 \, kg/m^3$

$\nu_m = \frac{\mu_m}{\rho_m} = 0.018$ stokes

$$\frac{L_p}{L_m} = \frac{100}{1}$$

$$\frac{300}{100} = L_m$$

$$L_m = 3m$$

By Reynold's model law.

$$\frac{\rho_p V_p L_p}{\mu_p} = \frac{\rho_m V_m L_m}{\mu_m}$$

$$\frac{V_p \times 300}{0.012} = \frac{30 \times 3}{0.018}$$

$$V_p = 0.2 \, m/s$$

$$\frac{F_p}{F_m} = \frac{(\rho L^2 V^2)_p}{(\rho L^2 V^2)_m}$$

$$= \frac{1030 \times 300^2 \times 0.2^2}{1.24 \times 3^2 \times 30^2}$$

$$\frac{F_p}{F_m} = 369.18 \, N$$

$$F_p = 369.18 \times 60$$

$$= 22150.53 \, N$$

Froude's Model Law

"The Froude number for the prototype and model should be equal".

It includes

(1) free surface flows such as flow over spillways, weirs, sluice, etc

(2) flow of jet from an orifice or nozzle

(3) where waves are likely to be formed.

(4) where fluids of different densities flow over one another.

According to Froude's Law

$$\frac{V_p}{\sqrt{g_p L_p}} = \frac{V_m}{\sqrt{g_m L_m}}$$

If tests are conducted on the same place where prototype is to be operated then $g_p = g_m$.

$$\frac{V_p}{\sqrt{L_p}} = \frac{V_m}{\sqrt{L_m}}$$

$$\frac{V_p}{V_m} = \sqrt{\frac{L_p}{L_m}}$$

$$\boxed{\frac{V_p}{V_m} = \sqrt{L_r}}$$

Scale ratio for time.

$$\text{time} = \frac{\text{length}}{\text{velocity}}$$

$$T_r = \frac{T_p}{T_m} = \left[\frac{L}{V}\right]_p \times \left[\frac{V}{L}\right]_m = \frac{L_p}{L_m} \times \frac{V_m}{V_p}$$

$$= L_r \times \frac{1}{\sqrt{L_r}}$$

$$\boxed{T_r = \sqrt{L_r}}$$

Scale ratio for acceleration.

$$\text{acceleration} = \frac{\text{velocity}}{\text{Time}}$$

$$a_r = \frac{a_p}{a_m} = \frac{\left[\frac{V}{T}\right]_p}{\left[\frac{V}{T}\right]_m} = \frac{V_p}{T_p} \times \frac{T_m}{V_m}$$

$$= \sqrt{L_r} \times \frac{1}{\sqrt{L_r}} = 1.$$

Scale ratio for discharge.

$$Q_r = \frac{Q_p}{Q_m} = \frac{\left[\frac{L^3}{T}\right]_p}{\left[\frac{L^3}{T}\right]_m} = \frac{L^3_p}{L^3_m} \times \frac{T_m}{T_p} = L_r^3 \times \frac{1}{\sqrt{L_r}} = L_r^{2.5}.$$

Scale ratio for force.

$$F_r = \frac{F_p}{F_m} = \frac{\rho_p L_p^2 V_p^2}{\rho_m L_m^2 V_m^2} = \left(\frac{\rho_p}{\rho_m}\right) \times L_r^2 \times (\sqrt{L_r})^2 \Rightarrow$$

if density of fluid in model and prototype.

$$L_r^2 \cdot L_r = \underline{\underline{L_r^3.}}$$

Scale ratio for pressure intensity.

$$\text{pressure intensity} = p = \frac{\text{Force}}{\text{Area}} = \frac{\rho L^2 V^2}{L^2} = \rho V^2.$$

$$p_r = \frac{p_p}{p_m} = \frac{\rho_p V_p^2}{\rho_m V_m^2}$$

if fluid is same, $\rho_p = \rho_m$

$$p_r = \frac{V_p^2}{V_m^2} = L_r.$$

Scale ratio for torque.

Torque = force × displacement.

$$T_r = \frac{T_p}{T_m} = \frac{F_p \times L_p}{F_m \times L_m} = F_r \times L_r = L_r^3 \times L_r$$

$$= \underline{\underline{L_r^4}}$$

Scale ratio for Power = $F \times$ velocity

$$= \frac{F \times L}{T}$$

$$P_r = \frac{P_p}{P_m} = \frac{F_p \times L_p}{T_p} \times \frac{T_m}{F_m \times L_m} = F_r \cdot L_r \cdot \frac{1}{T_p/T_m}$$

$$= \underline{L_r^3 \cdot L_r \cdot 1} = L_r^{3.5}$$

A 7.2m height and 15m long spillway discharges 94 m^3/s discharge under a head of 2m. If a 1:9 scale model of this spillway is to be constructed, determine the model dimentions, head over the spillway model and the model discharge. If model experiences a force of 7500N, Determine the force on the prototype.

Prototype

$h_p = 7.2m$

$L_p = 15m$

$Q_p = 94\ m^3/s$.

$H_p = 2m$

$F_p = ?$

Model

$F_m = 7500N$

$L_m = ?$

$h_m = ?$

$H_m = ?$

$Q_m = ?$

$$\frac{L_p}{L_m} = \frac{9}{1} = L_r$$

$$\frac{15}{9} = L_m$$

$$\underline{L_m = 1.67m}$$

$$\frac{h_p}{h_m} = L_r$$

$$7.2 = 9 \times h_m$$

$$\underline{h_m = 0.8m}.$$

$$\frac{H_p}{H_m} = 9$$

$$\frac{2}{H_m} = 9$$

$$H_m = \underline{0.222m}$$

$$\frac{F_p}{F_m} = F_r = L_r^3$$

$$F_p = 9^3 \times 7500$$

$$\underline{= 5467500\ N}$$

$$\frac{Q_p}{Q_m} = L_r^{2.5}$$

$$\frac{94}{9^{2.5}} = Q_m$$

$$Q_m = \underline{0.387}\ m^3/s.$$

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A spillway model is to be built to a geometrically similar scale of $\frac{1}{50}$ across a flume of 600mm width. The prototype is 15m high and max head on it is expected to be 1.5m. (i) What height of model and what head on the model should be used?

$$L_r = 50 = \frac{L_p}{L_m}$$

$B_p = 600\ mm$

$H_p = 15m$

$h_p = 1.5m$

$H_m = ?$

$h_m = ?$

$$\frac{H_p}{H_m} = L_r$$

$$\frac{15}{50} = H_m$$

$$H_m = 0.3\ m$$

$$\frac{h_p}{h_m} = L_r$$

$$\frac{1.5}{50} = \underline{0.03\ m}$$

Classification of Models:

Undistorted Models: These models are which are geometrically similar to their prototypes or in other words if the scale ratio for the linear dimensions of model and prototype is same, the model is called Undistorted model.

Distorted Models: A model is said to be distorted if it is not geometrically similar to its prototype. In a distorted model different scale ratios for the linear dimensions are adopted.

A ship model of scale $\frac{1}{50}$ is moved through sea water at a speed of 1m/s. A force of 2N is required to tow the model. Determine the speed of ship and the force on the ship. If prototype is subjected to wave resistance only.

Prototype

$V_p = ?$

$F_p = ?$

Model

$V_m = 1\, m/s$

$F_m = 2N$

$\frac{L_p}{L_m} = \frac{1}{50}$

$\frac{L_p}{L_m} = 50 = L_r$

$$\frac{V_p}{V_m} = \sqrt{L_r}$$

$$V_p = \sqrt{50} \times 1$$

$$= 7.071\, m/s.$$

$$\frac{F_p}{F_m} = L_r^3$$

$$F_p = 50^3 \times 2$$

$$= 250000N$$

A 1:64 model is constructed of an open channel in concrete which has Manning's N = 0.014. Find the value of N for the model.

$\frac{L_p}{L_m} = 64$

$N_p = 0.014$

$N_m = ?$

$$V = \frac{1}{N} m^{2/3} \cdot i^{1/2}$$

'ip and im are dimensionless

$$\frac{V_p}{V_m} = \frac{N_m}{N_p} \times (L_r)^{2/3} \quad (1)$$

$$\sqrt{L_r} = \frac{N_m}{0.014} \times (L_r)^{2/3}$$

$$N_m = 0.007$$

The characteristics of the spillway are to be studied by means of a geometrically similar model constructed to the scale ratio of 1:10.

(1) If the max rate of flow in the prototype is 28.3 m³/s. what will be the corresponding flow in model?

(2) If the measured velocity in the model at a point on the spillway is 2.4 m/s, what will be the corresponding value in prototype

(3) If the hydraulic jump at the foot of the model is 50mm high, what will be the height of jump in prototype.

(4) If the energy dissipated per sec in the model is 3.5 Nm what energy is dissipated per sec in the prototype.

Soln $L_r = 10$.

$Q_p = 28.3\ m^3/s$.

$Q_m = ?$

(1) $\dfrac{Q_p}{Q_m} = L_r^{2.5}$

$\dfrac{28.3}{10^{2.5}} = Q_m$

$Q_m = \underline{0.0895\ m^3/s}$.

(2) $V_m = 2.4\ m/s$.

$V_p = ?$

$\dfrac{V_p}{V_m} = \sqrt{L_r}$

$V_p = \sqrt{10} \times 2.4$

$= \underline{7.589\ m/s}$

(3) $H_m = 50mm$

$H_p = ?$

$\dfrac{H_p}{H_m} = L_r$

$H_p = 10 \times 50$

$= 500mm$

(4) Energy dissipated/s in model = 3.5 Nm.

$$E_r = \frac{E_p}{E_m} = \frac{(\text{Work done})/\text{Time})_p}{(\text{Work done})/\text{Time})_m}$$

$$= \frac{\rho_p L_p^2 V_p^2 \cdot L_p}{\rho_m L_m^2 V_m^2 \cdot L_m} \times \frac{T_m}{T_p}$$

$$= L_r^2 \cdot L_r \cdot L_r \times \frac{1}{\sqrt{L_r}}$$

$$= L^4 \times L^{-1/2}$$

$$= L^{7/2} = \underline{\underline{L^{3.5}}}$$

$$E_p = 10^{3.5} \times 3.5$$

$$= \underline{11067.9\ N}$$

Show by Dimensional analysis, that the power P developed by a hydraulic turbine is given by.

$$P = \rho N^3 D^5 f\left[\frac{N^2 D^2}{gH}\right]$$

where ρ = density, N = rotational speed, D = dia of runner, H = working head and g = accln due to gravity

Soln : Power is a function of
(a) Density, (b) Rotational speed (c) Dia of runner
(d) working head (e) accln due to gravity

Dimensions :

Power = ML^2T^{-3}

Rotational Speed = T^{-1} $\rho = ML^{-3}$

Dia of runner = L

Working head = L

g = LT^{-2}

By Rayleigh's method

$$P = f[\rho, N, D, H, g]$$

$$P = k \cdot \rho^a \cdot N^b \cdot D^c \cdot H^d \cdot g^e$$

$$ML^2T^{-3} = k \cdot (ML^{-3})^a \cdot [T^{-1}]^b \cdot [L]^c [L]^d [LT^{-2}]^e$$

equating the powers

M ; $1 = a$

L ; $2 = -3a + c + d + e \implies c = 3a - d - e + 2 \implies 5 - d - e$

T ; $-3 = -b - 2e \implies b = 3 - 2e$

$$P = k \cdot \rho^1 \cdot N^{3-2e} \cdot D^{5-d-e} \cdot H^d \cdot g^e$$

$$P = k\rho \cdot \frac{N^3}{N^{2e}} \cdot \frac{D^5}{D^d D^e} \cdot H^d \cdot g^e$$

~~$P = k\rho N^3 D^5 \left[\frac{gH}{N^2 D^2}\right]^e$~~ $P = k \cdot \rho N^3 D^5 \left[\left(\frac{H}{D}\right)^d \cdot \left(\frac{g}{D \cdot N^2}\right)^e\right]$

$$\boxed{P = k \cdot \rho N^3 D^5 f\left[\frac{D^2 N^2}{gH}\right]}$$

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7/03/2017

model - 2

uniform flow in a open channel hydra

Imp
1. Explain the classification of the flow in open channel

The different classification of flow channels are.

1. Flow steady and unsteady flow
2. Uniform and Non uniform flow
3. Laminar and Turbulent flow
4. Subcritical, critical and Supercritical flow

① Steady flow and unsteady flow

(i) Steady flow: If flow characteristics Such as velocity and rate of flow at any point in the open channel flow does not change with respect to the time. then that flow is Said to be Steady flow.

mathematically $\frac{dv}{dt} = 0$ $\frac{dQ}{dt} = 0$

(ii) unsteady flow :- If flow characteristics Such as velocity and rate of flow at any point in the open channel flow changes with respect to time then that flow is Said to be unsteady flow.

mathematically $\frac{dw}{dt} \neq 0 \quad \frac{d\theta}{dt} \neq 0$

② uniform flow and Non uniform flow

(i) uniform flow :- For a given length of a channel the depth of the flow, slope of the channel and the cross sectional area remains constant, then that type of flow is said to be uniform flow.

mathematically it is expressed as

$$\frac{dy}{ds} = 0 \qquad \frac{\partial v}{\partial s} = 0$$

(ii) Non uniform flow :- For a given length of a channel the depth of the flow, slope of the channel and the cross sectional area will change, then that type of flow is Said to be Non - uniform flow.

mathematically it is expressed as

$$\frac{\partial y}{ds} \neq 0 \qquad \frac{dV}{\partial s} \neq 0$$

③ Laminar flow and turbulent flow :

(i) Laminar flow :- The flow in a open channel is said to be laminar flow, if the Reynould's number is less than 500 or 600 and it is given by

$$Re = \frac{\rho V D}{\mu} < 500 \text{ or } 600$$

where D is hydrallic mean depth

$$D = \frac{A}{P} = \frac{\text{wetted area}}{\text{wetted perimeter}}$$

ii) Turbulent flow:- If the Renould's number is more than 2000, then that type of flow is called as turbulent flow.

$$Re > 2000$$

(4) Subcritical, critical and super critical flow

(i) Subcritical :- The fluid flow in open channel is said to be subcritical, if the Froude's number is less than 1. It is also called as streaming flow.

$$\boxed{F_e = \frac{V}{\sqrt{Lg}} < 1}$$

(ii) critical flow:- If the flow is said to be critical flow, then Froude's number is equal to '1'.

$$\boxed{F_e = \frac{V}{\sqrt{Lg}} = 1}$$

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(iii) Super critical flow :- The flow in which the channel is said to be Super critical, if the Froude's number is greater than 1.

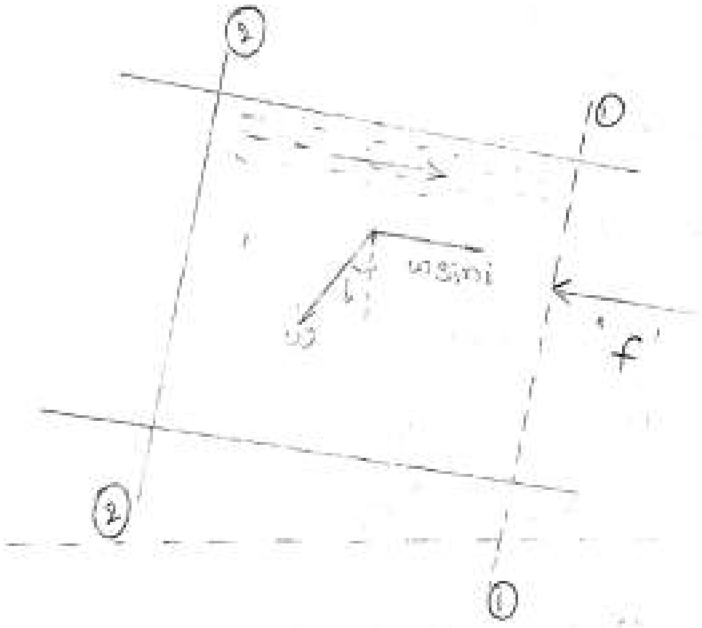
$$\boxed{F_e = \frac{V}{\sqrt{Lg}} > 1}$$

IMP Derive the chezy's equation for uniform flow in open channel with usual notation

~~D~~erive an equation for discharge in a open channel for chezy's formula.

or

Discharge in a open channel using chezy's formula



consider a uniform flow of water in a channel as shown in the figure. As the flow is uniform it means that the velocity, area of flow, depth will be constant for a given length of a channel.

Consider a section 1 - 1 and 2 - 2 as shown in figure where

L = Length of the channel

A = Area of the channel

i = slope of the bed

V = mean velocity of flow of water

P = wetted perimeter of the cross section

f = Frictional resistance per unit velocity per unit area.

m = hydraulic mean depth

The weight of the water at section 1 - 1 and 2 - 2 is given by

$$W = \text{Specific weight} \times \text{Volume}$$

$$= w \times A \times L$$

Resolving the component of W along the direction of flow will give $W \sin i$.

$$W = wA \times L \times \sin i$$

The frictional resistance against the motion of water is equal to

$$f \times \text{Surface area} \times (V^n)$$

The value of 'n' is found experimentally is 2 and surface area

Surface area = $P \times L$

i.e the frictional resistance = $f \times P \times L \times V^2$

The different forces acting on the water along the flow direction is given by

(i) component of wait of the water along flow direction

(ii) Frictional resistance along the flow of water

In case of uniform flow, the velocity of the flow is constant for a given length of a channel. Hence there is no acceleration acting on the water.

So that the resultant force acting in the direction of flow should be zero.

Resolving all the forces in the direction of the flow.

$$w \times A \times L \sin i - f \times P \times L \times V^2 = 0$$

$$w \times A \times L \sin i = f \times P \times L \times V^2$$

$$V^2 = \frac{w \times A \times \sin i}{f \times P}$$

$$V^2 = \frac{w \times A \times i}{f \times P}$$

$$V = \sqrt{\frac{w}{f} \times \frac{A}{P} \times i}$$

$\sqrt{\frac{w}{f}} = c$, $\frac{A}{P} = m \rightarrow$ Hydraullic mean depth

chezy's constant

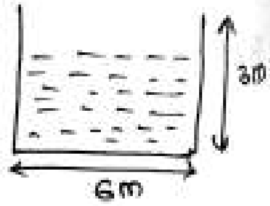
$$\boxed{V = C \times \sqrt{mi}}$$

$$Q = A \times V$$

$$\boxed{Q = A \times c\sqrt{mi}}$$ Discharge.

problems :-

① Find the velocity of flow and the rate of flow of water in a rectangular channel of 6m width and 3m deep where it is running full, the channel is having bed slope as 1 in 2000 take chezy's constant 'C' as 55.



Sol^n:

$A = 6 \times 3 = 18 m^2$

$i = \frac{1}{2000}$

$C = 55$

$V = ?$

$Q = ?$

$V = C\sqrt{mi}$

$Q = A \times V$

$m = \frac{A}{P}$

$P = d + b + d$

$P = b + 2d$

$= 6 + (2 \times 3)$

$\boxed{P = 12 m}$

$m = \frac{A}{P}$

$= \frac{18}{12}$

$\boxed{m = 1.5 m}$

$V = C\sqrt{mi}$

$= 55\sqrt{1.5 \times \left(\frac{1}{2000}\right)}$

$= 55\sqrt{1.5 \times 0.0005}$

$\boxed{V = 1.5062 \ m/s}$

$Q = A \times V$

$= 18 \times 1.5062$

$\boxed{Q = 27.11 \ m^3/sec}$

(2) Find the slope of the bed of a rectangular channel of width 5 m, depth of water is 2 m and the rate of flow is given as 20 m^3/s. Take chezy's constant C = 50.

Soln

$A = 5 \times 2 = 10 \ m^2$

$Q = 20 \ m^3/s$

$C = 50$

$i = ?$

$m = \frac{A}{P}$

$P = d + b + d$

$P = b + 2d$

$= 5 + (2 \times 2)$

$= 5 + 4$

$\boxed{P = 9}$

$m = \frac{A}{P}$

$= \frac{10}{9}$

$\boxed{m = 1.1111 \ m}$

$Q =$

$V = C \times \sqrt{mi}$

$Q = 50 \times \sqrt{i \times 1.1111}$

$\frac{Q}{50} = \sqrt{i \times 1.1111}$

$\left(\frac{Q}{50}\right)^2 = i \times 1.1111$

$i = \left(\frac{Q}{50}\right)^2 \times \frac{1}{1.1111}$

$i = 1.4400 \times 10^{-3}$

$\boxed{i = 0.00144}$

$i = \frac{1}{694.45}$

$\boxed{i = 1 \text{ in } 694.45}$

(3) A flow of water of 100 lit/sec flows down in a rectangular flue of width 600 mm and having adjustable water slope. Take C = 56. Find the bottom slope necessary for the uniform flow with a depth of flow 300 mm. Also find conveyence 'K' of the flow.

Sol^n

$w = 600 \text{ mm} = 600 \times 10^{-3} \text{ m} = 0.6 \text{ m}$

$d = 300 \text{ mm} = 300 \times 10^{-3} \text{ m} = 0.3 \text{ m}$

$A = 0.6 \times 0.3$

$= 0.18 \text{ m}^2$

$Q = 100 \ lit/sec = \frac{1000}{1000} = 0.1 \ m^3/sec$

$C = 56$

$P = d + b + d$

$= 0.3 + 0.6 + 0.3$

$= 1.2$

$m = \frac{A}{P}$

$= \frac{0.18}{1.2}$

$m = 0.15$

$Q = A . C\sqrt{mi}$

$0.1 = 0.18 \times 56 \sqrt{0.15 \times i}$

$0.1 = 10.08 \times \sqrt{0.15 \times i}$

$(9.9206 \times 10^{-3})^2 = 0.15 \times i$

$(0.0099)^2 = 0.15 \times i$

$\boxed{i = 6.56 \times 10^{-4}}$

$i = 1$ in 6.56

$\boxed{i = 1 \text{ in } 1524.09}$

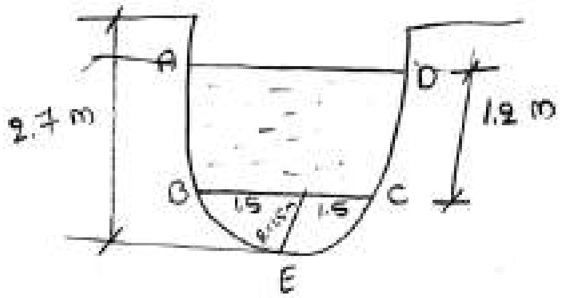
$Q = K\sqrt{i}$

$K = AC\sqrt{m}$

$K = 0.18 \times 56 \times \sqrt{0.15}$

$\boxed{K = 3.90 \ m^3/sec}$

④ Find the discharge of water through a channel shown in figure. take ~~below~~ $C = 60$ and $i = 1$ in 2000



Given :-

Area = ABCD + BEC

Area of perimeter $= 1.2 \times 3$

$= 3.6$ m

Area of semi circle $= \dfrac{\pi r^2}{2} = \dfrac{\pi \times 1.5^2}{2}$

$= 3.53$ m

Total Area $= 3.6 + 3.53$

$= 7.13\ m^2$

$P = 2DC + \pi r$

$= 2 \times 1.2 + \pi \times 1.5$

$\boxed{P = 7.11 \text{ m}}$

$m = \dfrac{A}{P} = \dfrac{7.13}{7.11}$

$\boxed{m = 1.003 \text{ m}}$

$$Q = Ac\sqrt{mi}$$

$$Q = 7.13 \times 60 \sqrt{1.003 \times \frac{1}{2000}}$$

$$\boxed{Q = 9.58 \ m^3/sec}$$

(5) Find the discharge of trapezoidal channel of a bed width 8m and side slope 1 horizontal to 3 vertical. The depth of flow of water is 2.4 m, C = 50. The slope of bed of channel is 1 in 4000.

Sol

$w = 8m$

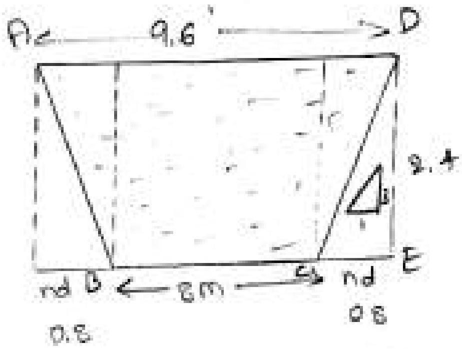
$d = 2.4 \ m$

$A = 8 \times 2.4$

$=$

$C = 50$

$i = \frac{1}{4000}$



$nd = \frac{1}{3} \times 2.4$

$$\boxed{nd = 0.8 \ m}$$

Top width

$8 + (2 \times 0.8)$

$= 9.6 \ m^2$

Total area = Area of rectangle + (2 × Area of triangle)

Area of rectangle BCEF $= 8 \times 2.4$

$= 19.2$

Area of triangle $= \cancel{2} \times \frac{1}{\cancel{2}} \times 0.8 \times 2.4$

$= 1.92$

Total area = 19.2 + 1.92

$$\boxed{A = 21.12\ m^2}$$

$DC = \sqrt{(0.8)^2 + (2.4 ...}$

$DC = 2.529$

$P = AB + BC + DC$

$= 2AB + BC$

$= (2 \times 2.529) + 8$

$$\boxed{P = 13.05\ m}$$

$m = \frac{P}{A}$

$= \frac{13.05}{21.12}$

$m = \frac{A}{P} = \frac{21.12}{13.05}$

$$\boxed{m = 1.618}$$

$Q = A \times C\sqrt{mi}$

$= 21.12 \times 50\sqrt{1.618 \times \frac{1}{4000}}$

$$\boxed{Q = 21.23\ m^3/sec}$$

(6) Find the bed slope of a trapezoidal cha... section of a bed width 6m, depth of the water is 3m and the side slope is 3 horizontal to 4 vertical when the discharg... through the channel is 30 m^3/s. Take C = ...

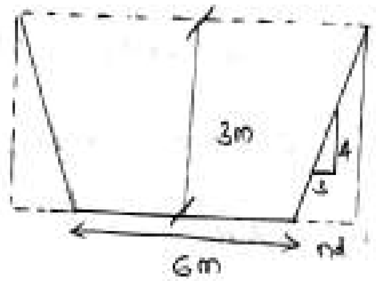
$i = ?$

$b = 6m$

$d = 3m$

$Q = 30\ m^3/s$

$C = 70$



$nd = \frac{3}{4} \times 3$

$\boxed{nd = 2.25\ m}$

Top width $= 6 + 2.25 + 2.25$

$= 10.5\ m$

$A = A$ of $\square^{le} + 2\ [A$ of $\Delta^{le}]$

$A = 6 \times 3 + 2\left[\frac{1}{2} \times 2.25 \times 3\right]$

$= 18 + 6.75$

$\boxed{A = 24.75\ m^2}$

$DC^2 = (2.25)^2 + 3^2$

$DC = \sqrt{(2.25)^2 + 3^2}$

$\boxed{DC = 3.75}$

$P = \overset{3.75}{2} + 3.75 + 6$

$\boxed{P = 13.5\ m}$

$m = \frac{A}{P}$

$= \frac{24.75}{13.5}$

$\boxed{m = 1.83\ m}$

$Q = A \times C \times \sqrt{mi}$

$30 = 24.75 \times 70 \times \sqrt{1.83 \times i}$

$30 = 1732.5\ \sqrt{1.83 \times i}$

$0.0173 = \sqrt{1.83 \times i}$

$2.9929 \times 10^{-4} = 1.83 \times i$

$\boxed{i = 1.6354 \times 10^{-4}}$

$i = 1$ in

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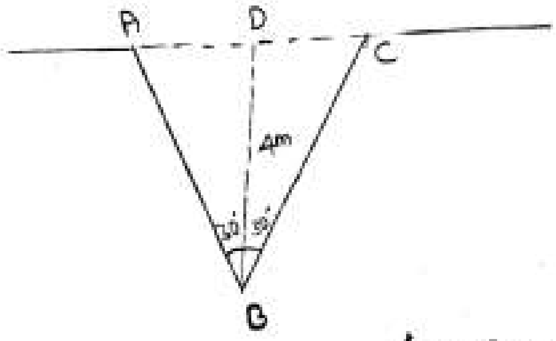
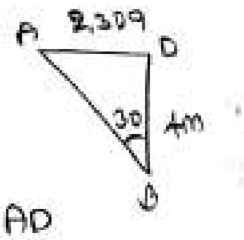
(7) Find the rate of flow of water through a 60° V-noth channel as shown in figure take C = 55 and slope bed = 1/2000 and depth = 4 m





Sol^n Given

$C = 55$

$i = \frac{1}{2000}$

$Q = ?$

$\tan 30 = \frac{AD}{4}$

$AD = \tan 30 \times 4$

$AD = 2.301 \text{ m}$

$AB = \sqrt{\underset{AD^2}{(2.3091)^2} + \underset{DB^2}{4^2}}$

$AB = 4.618 \text{ m}$

$A = 2 \times \left[\frac{1}{2} \times 2.309 \times 4\right]$

$\boxed{A = 9.237 \text{ m}^2}$

$P = AB + BC$

$P = 2 + 4.618$

$\boxed{P = 9.236 \text{ m}^2}$

$m = \frac{A}{P}$

$= \frac{9.236}{9.236}$

$\boxed{m = 1}$

$Q = A \times C \sqrt{mi}$

$Q = 9.237 \times 55 \sqrt{1 \times \frac{1}{2000}}$

$\boxed{Q = 11.360 \text{ m}^3/\text{sec}}$

(8) Find the dia of the circular pipe which is laid at the slope of 1 in 8000 and carrying a discharge of 800 lit/sec when flowing half full. Take mannings co-efficient 'N' = 0.02.

Mannings formula is given by $\boxed{C = \frac{1}{N} m^{1/6}}$

and discharge $\boxed{Q = AC\sqrt{mi}}$

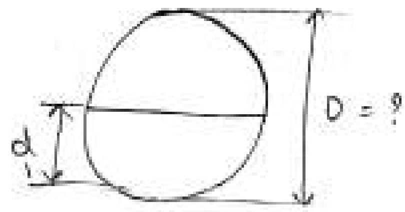
$i = 1/8000$

$Q = 800\ \text{lit/sec}$

$= \dfrac{800}{1000}$

$Q = 0.8\ m^3/sec$

$d = \dfrac{D}{2}$



$\cancel{A = -\frac{1}{2}\left[\frac{\pi D^2}{4}\right]}$

$\cancel{= -\frac{1}{2}\{\pi \times}$

$A = \dfrac{1}{2}\left[\dfrac{\pi D^2}{4}\right]$

$P = \dfrac{1}{2}\pi D$

$\boxed{A = \dfrac{\pi D^2}{8}}$

$\boxed{P = \dfrac{\pi D}{2}}$

$m = \dfrac{A}{P} = \dfrac{\frac{\pi D^2}{8}}{\frac{\pi D}{2}} = \dfrac{D}{4}$

$\boxed{m = \dfrac{D}{4}}$

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mannig's formula is given by

$$C = \frac{1}{N} m^{1/6}$$

$$C = \frac{1}{0.02} \times \left(\frac{D}{4}\right)^{1/6}$$

$$Q = AC\sqrt{mi}$$

$$0.8 = \frac{\pi D^2}{8} \times \frac{1}{0.02} \times \left(\frac{D}{4}\right)^{1/6} \sqrt{\frac{D}{4} \times \frac{1}{8000}}$$

$$0.8 = \frac{\pi \times D^2 \times D^{1/6}}{8 \times 0.02 \times 4} \times \frac{\sqrt{D}}{\sqrt{4 \times 8000}}$$

$$28.8488 = \pi D^2 \times D^{1/6} \times D^{1/2}$$

$$9.1828 = D^{2 + 1/6 + 1/2}$$

$$9.182869 = D^{8/3}$$

$$D = (9.182869)^{3/8}$$

$$\boxed{D = 2.2967 \text{ m}}$$

Most Economical channel Sections

A Section of channel is Said to be most economical when the cost of construction of channel is minimum.

* The cost of ~~co~~construction depends on the excavation and lying.
* To keep the cost of construction minium the wetted perimeter for the given discharge

should be minimum.

* This condition is utilized for the determining the dimension of the economical sections for different form of channel.

* most economical channels also called as efficient section or best section

For a given discharge, the section is economical when,

$$Q = AC\sqrt{mi}$$

$$Q = AC\sqrt{\frac{A}{P} \times i}$$

$$Q = K \times \sqrt{\frac{1}{P}} \qquad \boxed{K = AC\sqrt{Ai}}$$

$$\boxed{Q \propto \frac{1}{\sqrt{P}}}$$

Hence the discharge 'Q' will be maximum when the wetted perimeter should be minimum.

most economical rectangular channel section

For the condition for most economical rectangular section is that for the given area, the wetted periment should be minimum.

consider a rectangular section as shown in figure.

Let

A = Area of the channel

d = depth of the channel

b = width of the channel

p = perimeter of the channel

$$A = b \times d$$

$$b = \frac{A}{d} \rightarrow ①$$

$$P = b + 2d$$

Substituting eq^n in P

$$P = \frac{A}{d} + 2d \quad — ②$$

For the most economical section 'p' should be minimum i.e $\frac{dp}{d(d)} = 0$

Differentiating eq^n ③ wrt 'd'

$$\frac{dp}{dd} = \frac{-A}{d^2} + 2 = 0$$

$$-\frac{A}{d^2} + 2 = 0$$

$$\frac{A}{d} = 2$$

$$\boxed{A = 2d^2} \longrightarrow ③$$

Sub eq ③ in ①

$$b = \frac{2d^2}{d}$$

$$\boxed{b = 2d} \longrightarrow ④$$

$$m = \frac{A}{P} = \frac{2d^2}{2d+2d} = \frac{2d^{\cancel{2}}}{\cancel{4}_2 \cancel{d}}$$

$$\boxed{m = d/2}$$

The given rectangular channel will be more economical when

(i) $A = 2d^2$

(ii) $b = 2d$

(iii) $m = d/2$

① A rectangular channel carries a water at a rate of 400 lit/s when bed slope is 1 in 2000. Find the most economical dimension of the channel. Take C = 50

Sol^n $Q = 400$ lit/s $= 0.4\ m^3/s$

$$i = \frac{1}{2000}$$

$$C = 50$$

$$Q = AC\sqrt{mi}$$

$A = 2d^2$

$b = 2d$

$m = \frac{d}{2}$

$$0.4 = 2d^2 \times 50 \times \sqrt{\frac{d}{2} \times \frac{1}{2000}}$$

$$\frac{0.4}{50} = \frac{2d^2 \times \sqrt{d}}{\sqrt{2 \times 2000}}$$

$$\frac{0.4 \times \sqrt{2 \times 2000}}{50 \times 2} = d^{5/2}$$

$$\boxed{d = 0.577 \text{ m}}$$

$$A = 2d^2$$
$$= 2 \times (0.577)^2$$
$$A = 0.665 \text{ m}^2$$

$$b = 2d$$
$$= 2 \times 0.577$$
$$\boxed{b = 1.154 \text{ m}}$$

$$m = d/2$$
$$= \frac{0.577}{2}$$
$$\boxed{m = 0.288}$$

☆ Imp

(2) A rectangular channel 4m wide as a depth of water 1.5 m, i = 1 in 1000, C=55. It is desired to increase the discharge to maximum by changing the dimension of Section for the constant area of cross Section, Bed Slope and roughness of the channel. Find the new dimension of the channel and increase in discharge.

$h^3 \times n$

Given :-

$b = 4\,m$

$d = 1.5\,m$

$i = \frac{1}{1000}$

$C = 55$

$A = b \times d$

$\boxed{A = 6\,m^2}$

$m = \frac{A}{P}$

$P = b + 2d$

$= 4 + 2(1.5)$

$\boxed{P = 7\,m}$

$= \frac{6}{7}$

$\boxed{m = 0.857}$

$Q = A \times C\sqrt{m \times i}$

$= 6 \times 55\sqrt{0.857 \times \frac{1}{1000}}$

$\boxed{Q = 9.66\ m^3/sec}$

$b' =$ new width

$d' =$ new depth

$m' =$ new mean depth

$A = b' \times d'$

$6 = b' \times d'$

For most economical channel section

$b' = 2d'$

$m' = d'/2$

$A' = b' \times d'$

$6 = b' \times d'$

$6 = 2d' \times d'$

$6 = 2d'^2$

$d_1^2 = 3$

$\boxed{d_1 = 1.732\ m}$

$b' = 2d'$

$= 2 \times 1.732$

$\boxed{b' = 3.464\ m}$

$$m = \frac{d'}{2} = \frac{1.732}{2}$$

$$\boxed{m = 0.866}$$

$$Q = A \times C \sqrt{mi}$$

$$= 6 \times 55 \times \sqrt{0.866 \times \frac{1}{1000}}$$

$$\boxed{Q = 9.7111 \ m^3/sec}$$

$$Q_{in} = 9.71 - 9.66$$

$$\boxed{Q_{in} = 0.05} \ m^3/sec$$

Discharge for the economical Section

$$Q = AC \times \sqrt{mi}$$

$$A = b' \times d'$$

$$= 3.464 \times 1.732$$

$$A = 5.99$$

$$Q = 5.99 \times 55 \times \sqrt{0.866 \times \frac{1}{1000}}$$

$$\boxed{Q = 9.69 \ m^3/sec}$$

$$\approx$$

$$Q = 9.7 \ m^3/sec$$

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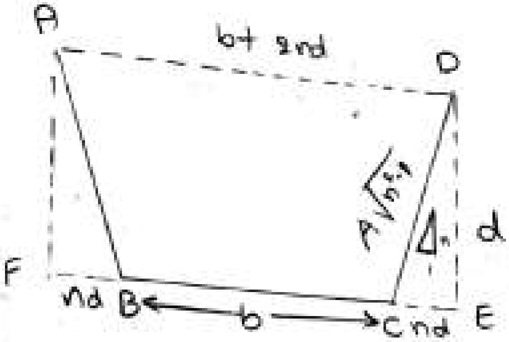
most Economical trapezoidal channel section



Trapezoidal channel will be most economical when the wetted perimeter is minimum.

Consider a trapezoidal channel as shown in figure.

Let

b = Bottom width of the channel

d = depth of the channel

i = slope of the channel

1/n = side slope of the channel

Area of the channel

$$A = \left(\frac{BC + AD}{2}\right) \times d$$

$$A = \left[\frac{b + (b+2nd)}{2}\right] \times d$$

$$A = \left[\frac{2b + 2nd}{2}\right] \times d$$

$$= \frac{\cancel{2}(b+nd)}{\cancel{2}} \times d$$

$$A = (b+nd] \times d \quad \text{——} \; 1$$

$$\frac{A}{d} = b + nd$$

$$\boxed{b = \frac{A}{d} - nd} \quad \text{—— (2)}$$

$$P = AB + BC + DC$$

$$DC^2 = CE^2 + DE^2$$

$$P = BC + 2AB$$

$$DC = \sqrt{n^2d^2 + d^2}$$

$$P = b + 2d\sqrt{n^2+1} \quad \text{—— (3)}$$

$$DC = d\sqrt{n^2+1}$$

$$P = \frac{A}{d} - nd + 2d\sqrt{n^2+1} \quad \text{—— (4)}$$

Since for the most economical section the perimeter should be minimum

$$\frac{dp}{d(d)} = 0$$

diff eq^n (4) wrt d

$$\frac{dp}{d(d)} = -\frac{A}{d^2} - n + 2\sqrt{n^2+1} = 0$$

$$-\frac{A}{d^2} - n + 2\sqrt{n^2+1} = 0$$

$$2\sqrt{n^2+1} = \frac{A}{d^2} + n$$

Sub A from eq^n (1)

$$2\sqrt{n^2+1} = \frac{(b+nd)d}{d^2} + n$$

$$2\sqrt{n^2+1} = \frac{b+nd}{d} + n$$

$$2\sqrt{n^2+1} = \frac{b+nd+nd}{d}$$

$$2\sqrt{n^2+1} = \frac{2nd+b}{d}$$

$$\boxed{d\sqrt{n^2+1} = \frac{2nd+b}{2}} \longrightarrow (5)$$

From the figure $\frac{b+2nd}{2} = \frac{1}{2}$ of the top width

and $d\sqrt{n^2+1}$ = one side of sloping

i.e Equation 5 is the required condition for a trapezoidal Section to be most economical.

Hydraulic mean depth,

$$m = \frac{A}{P}$$

$$= \frac{(b+nd)\,d}{b+2d\sqrt{n^2+1}}$$

From eq^n (5) putti

$$b+2nd = 2d\sqrt{n^2+1}$$

$$m = \frac{(b+nd)\times d}{b+b+2nd}$$

$$m = \frac{(b+nd)\times d}{2b+2nd}$$

$$= \frac{\cancel{(b+nd)}}{2\cancel{(b+nd)}}\times d$$

$$\boxed{m = d/2}$$

Hence for a trapezoidal Section to be most economical the hydraulic mean depth is equal to half of the depth of flow.

$$A = [b + nd] \times d$$

$$P = b + 2d\sqrt{n^2+1}$$

$$\frac{b+2nd}{2} = 2d\sqrt{n^2+1}$$

$$P = \frac{A}{d} - nd + 2d\sqrt{n^2+1}$$

$$m = \frac{d}{2}$$

① A trapezoidal channel has a side slope of 1 horizontal to 2 vertical and a bed slope of 1 in 5000. Find the dimension of the Section if it is most economical and determine the discharge of most economical Section if $C = 50$, $A = 40\ m^2$

Sol^n

$C = 50$

$A = 40\ m^2$ $\quad$ $b = ?$

$i = \frac{1}{5000}$ $\quad$ $d = ?$

$\quad$ $Q = ?$

$n = \frac{1}{2}$

$$\frac{b + 2nd}{2} = d\sqrt{n^2+1}$$

$$\frac{b + \cancel{2} \times \frac{1}{\cancel{2}} \times d}{2} = d\sqrt{\left(\frac{1}{2}\right)^2+1}$$

$$\frac{b+d}{2} = d\sqrt{\frac{1}{4}+1}$$

$$\frac{b+d}{2} = d\sqrt{\frac{5}{4}}$$

$$\frac{b+d}{2} = d \times 1.118$$

$$b+d = 2 \times 1.118 \times d$$

$$b+d = 2.236\ d$$

$$b = 2.236d - d$$

$$\boxed{b = 1.236 \times d} \longrightarrow (1)$$

$$A = [b + nd] \times d$$

$$40 = (1.236 \times d + \left(\frac{1}{2}\right) d]\ d$$

$$40 = 1.236\ d^2 + 0.5 d^2$$

$$40 = 1.736\ d^2$$

$$23.041 = d^2$$

$$\boxed{d = 4.8\ m}$$

$$b = 1.236 \times d$$

$$= 1.236 \times 4.8$$

$$\boxed{b = 5.9328}$$

$P = b + 2d\sqrt{n^2+1}$

$= 5.932 + (2 \times \tfrac{1}{2} \times 4.8)\sqrt{(\tfrac{1}{2})^2+1}$

$\boxed{P = 11.291\ m}$

$m = \dfrac{d}{2}$

$=$

$P = b + 2d\sqrt{n^2+1}$

$= 5.932 + (2 \times 4.8)\sqrt{(\tfrac{1}{2})^2+1}$

$\boxed{P = 16.66\ m}$

$m = \dfrac{d}{2} = \dfrac{4.8}{2} = 2.4$

$\boxed{m = 2.4\ m}$

$Q = AC\sqrt{mi}$

$= 40 \times 50 \times \sqrt{2.4 \times \left(\dfrac{1}{5000}\right)}$

$\boxed{Q = 43.81\ m^3/sec}$

(2) Design a most economical section for discharge of 50 m^3/s having a side slope of 2 horizontal to 1 Vertical and bed slope of 1 in 800. Take $C = 60$.

Sol^n

$Q = 50\ m^3/s$

$c = 60$

$i = \frac{1}{800}$

$n = \frac{2}{1} = 2$

$$\frac{b + 2nd}{2} = d\sqrt{n^2+1}$$

$$\frac{b + 2(2)d}{2} = d\sqrt{(2)^2+1}$$

$$\frac{b + \cancel{4}^{2}d}{\cancel{2}} = d\sqrt{5}$$

$$\frac{b+4d}{2} = d\sqrt{5}$$

$$b+4d = 2d\sqrt{5}$$

$$b+4d = 4.472\,d$$

$$\boxed{b = 0.472\,d} \longrightarrow 1$$

$$A = (b+nd)\,d$$

$$A = (0.472d + 2d)\,d$$

$$A = 0.472d^2 + 2d^2$$

$$\boxed{A = 2.47 \times d^2} \longrightarrow 2$$

$$Q = AC\sqrt{mi}$$

$$= 2.47d^2 \times 60 \times \sqrt{\frac{d}{2} \times \frac{1}{800}}$$

$$= 2.47d^2 \times 60 \times \sqrt{\frac{d}{\quad}}$$

$$= 2.47 d^2 \times 60 \times d^{1/2} \times 0.5^{1/2} \times (1.25 \times 10^{-3})^{1/2}$$

$$= 2.47 d^{5/2} \times 60 \times 0.707 \times 0.0353$$

$$50 = 2.47 \, d^{5/2} \times 1.497$$

$$33.4 = 2.47 \, d^{5/2}$$

$$13.522 = d^{5/2}$$

$$\boxed{d = 2.833 \text{ m}}$$

$$m = \frac{d}{2} = \frac{2.833}{2}$$

$$\boxed{m = 1.4165}$$

$$A = 2.47 \times d^2$$

$$A = 2.47 \times (2.83)^2$$

$$\boxed{A = 19.823 \text{ m}^2}$$

$$b = 0.472 \times d$$

$$= 0.472 \times 2.833$$

$$\boxed{b = 1.337 \text{ m}}$$

☆

(3) A trapezoidal channel has a side slope of 1:1 it is required to discharg $13.75 \text{ m}^3/\text{s}$ with the bed gradient of 1 in 1000. If unlined value of chezy's constant C = 44 and if lined with concrete its value is 60.

The cost per m^3 of excavation is 4 times the cost per m^2 of laying. If the channel is to be most efficient one find whether the lined cannel or unlined cannel will be cheaper. what will be dimension of the economical ~~channel~~ canal.

Sol^n

$Q = 13.75 \ m^3/sec$

$i = \frac{1}{1000}$

$n = 1$

lined	unlined
$c = 60$	$c = 44$

$x \rightarrow$ lining

c of $E = 4x$

cost per meter^3 of excavation is equal to 4 times the cost per m^2 of lining

Let the cost ~~cost~~ per m^2 of lining $= x$ then cost per m^2 of lining $= x$

For most economical channel

$$\frac{b + 2nd}{2} = d\sqrt{n^2+1}$$

$$\frac{b+2d}{2} = d\sqrt{2}$$

$$b + 2d = 2d\sqrt{2}$$

$$b = 2d\sqrt{2} - 2d$$

$$b = d\left[2 \times \sqrt{2} - 2\right]$$

$$\boxed{b = 0.828d} \longrightarrow (1)$$

$A = (b + nd)d$

$= (0.828d + d)d$

$$\boxed{A = 1.828\,d^2}$$

$Q = AC\sqrt{m \times i}$

<u>case (i)</u> :- For <u>unlined</u>

$C = 44$

$i = 1/1000$

$Q = 13.75$

$m = d/2$

$Q = AC\sqrt{m \times i}$

$13.75 = 1.828\,d^2 \times 44 \sqrt{\frac{d}{2} \times \frac{1}{1000}}$

$13.75 = 1.828\,d^2 \times 44 \frac{\sqrt{d}}{\sqrt{2 \times 1000}}$

$\frac{13.75\sqrt{2 \times 1000}}{1.828 \times 44} = d^{5/2}$

$d^{5/2} = 7.645$

$$\boxed{d = 2.256\text{ m}}$$

$b = 0.828\,d$

$= 0.828 \times 2.25$

$$\boxed{b = 1.863\text{ m}}$$

$m = d/2$

$= \frac{2.256}{2}$

$$\boxed{m = 1.128}$$

The cost of excavation per running length meter in unlined water channel is equal to

CE = Volume of channel × cost / m^3 of excavation

$= A \times L \times 4x$

$= (b+nd)d \times 1 \times 4x$

$= (1.86 + (1)\,2.257)\,2.257 \times 1 \times 4x$

$$\boxed{C.E = 37.16\,x}$$

case (ii) For lined

$C = 60$

$i = \frac{1}{1000}$

$Q = 13.75$

$m = d/2$

$Q = AC\sqrt{m \times i}$

$13.75 = 1.828\,d^2 \times 60\sqrt{\frac{d}{2} \times \frac{1}{1000}}$

$13.75 = 1.828\,d^2 \times 60 \;\frac{\sqrt{d}}{\sqrt{2 \times 1000}}$

$\frac{13.75 \times \sqrt{2 \times 1000}}{1.828 \times 60} = d^{5/2}$

$d^{5/2} = 5.606$

$$\boxed{d = 1.99\text{ m}}$$

$b = 0.828\,d$

$= 0.828 \times 1.99$

$$\boxed{b = 1.647\text{ m}}$$

$m = d/2 = \frac{1.99}{2}$

$$\boxed{m = 0.995}$$

CE = volume of channel × cost /m^3 of excavation

$$= A \times L \times 4x$$

$$= (b+nd)d \times 1 \times 4x$$

$$= (1.647 + ((1) \times 1.99))\, 1.99 \times 4x$$

$$\boxed{C.E = 29.0\, x}$$

The cost of lining is equal to perimenta of laying × cost of m^2 of laying

$$C \text{ for lin} = P \times \text{cost of } A/m^2$$

$$= \left(b + 2d\sqrt{n^2+1}\right) \times x$$

$$= \left(1.65 + (2 \times 1.99)\sqrt{1^2+1}\right) x$$

$$= 7.278$$

$$\approx$$

$$\boxed{C \text{ for lining} = 7.30\, x}$$

Total cost for lined channel

$$29x + 7.3x$$

$$36.3x$$

4) The trapezoidal channel with a side slow of 3 horizontal to 2 vertical has to be design to convey 10 m^3/s at a velocity of 1.5 m/s so that the amount of concrete for laying beds and sides is minimum. Find the wetted perimeter and the slope of the bed. if n = 0.14.

Sol^n:

$V = 1.5$ m/s

$Q = 10$ m^3/s

$N = 0.014$

$n = \frac{3}{2}$

$P = ?$

$i = ?$

$$\frac{b + 2nd}{2} = d\sqrt{n^2+1}$$

$$\frac{b + \cancel{2}\left(\frac{3}{\cancel{2}}\right)d}{2} = d\sqrt{\left(\frac{3}{2}\right)^2+1}$$

$$\frac{b+3d}{2} = d\sqrt{3.25}$$

$$\frac{b+3d}{2} = d\ 1.8027$$

$$b+3d = 3.6055\ d$$

$$b = 3.6055d - 3d$$

$$\boxed{b = 0.6055d\ m}$$

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$A = (b + nd)d$

$A = \left(0.605d + \left(\frac{3}{2} \times d\right)\right)d$

$\boxed{A = 2.105d^2}$

$Q = A \times V$

$10 = 2.105d^2 \, V$

$A = \frac{Q}{V} = \frac{10}{1.5} = 6.66 \ m^2$

$Q = A \times V$ $\boxed{A = 6.66 \ m^2}$

$10 =$

$Q = A \times V$

$10 = A \times 1.5$

$\boxed{A = 6.66 \ m^2}$

$6.666 = 2.105 \, d^2$

$3.1670 = d^2$

$\boxed{d = 1.779 \ m}$

$b = 0.605 \, d$

$= 0.605 \times 1.779$

$\boxed{b = 1.076 \ m}$

$m = \frac{d}{2} = \frac{1.779}{2} = 0.8895 \ m.$

$$C = \frac{1}{N} m^{1/6}$$

$$= \frac{1}{0.014} (0.8895)^{1/6}$$

$$\boxed{C = 70.04}$$

$$Q = AC\sqrt{mi}$$

$$10 = 6.666 \times 70.04 \sqrt{0.8895\, i}$$

$$10 = 466.88 \sqrt{0.8895\, i}$$

$$0.0227 = \sqrt{i}$$

$$i = 5.159 \times 10^{-4}$$

$$i = 1 \text{ in } 1938.25$$

<u>most economical circular channel section</u>:-

In case of circular channel the area of flow cannot be maintained constant with the change of depth of flow in a circular channel of any radius and also wetted area and wetted perimeter changes.

So that thus in case of circular channel for the most economical circular section 2 conditions must be for max

i) condition for maximum velocity

(ii) condition for maximum discharge

(i) Condition for maximum velocity :-

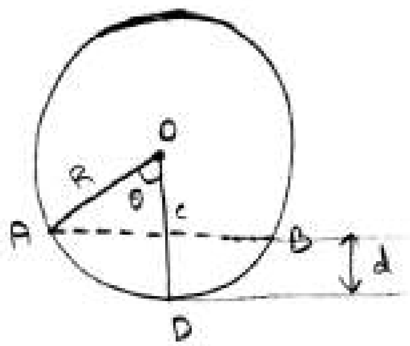


Figure shows a circular channel where water is flowing. Let 'd' is the depth of the water

2θ = angle suspended at centre by any water surface.

R = radius of the circle

In case of circular pipe the variable is only 'θ' hence for maximum value of $\frac{A}{P}$ we have a condition that

$$\frac{d\left(\frac{A}{P}\right)}{d\theta} = 0$$

The wetted area of the circular section is given by $A = R^2\left[\theta - \frac{\sin 2\theta}{2}\right]$

The wetted perimeter of the circular section is given by

$$P = 2R\theta$$

$$\frac{d\left(\frac{A}{P}\right)}{d\theta} = 0$$

$$\frac{P \times \frac{dA}{d\theta} - A \times \frac{dp}{d\theta}}{p^2} = 0$$

$$P \times \frac{dA}{d\theta} - A \times \frac{dp}{d\theta} = 0 \quad \text{—— (1)}$$

$$A = R^2\left[\theta - \frac{\sin 2\theta}{2}\right]$$

$$\frac{dA}{d\theta} = R^2\left[1 - \frac{\cos 2\theta \times \cancel{2}}{\cancel{2}}\right]$$

$$\frac{dA}{d\theta} = R^2\left[1 - \cos 2\theta\right] \quad \text{—— (2)}$$

$$\frac{dA}{d\theta} \cancel{=} \cancel{R^2} \qquad P = 2R\theta$$

$$\boxed{\frac{dp}{d\theta} = 2R} \rightarrow (3)$$

Substitute (2) and (3) in eqⁿ (1)

$$2R\theta\left[R^2(1-\cos 2\theta)\right] - R^2\left[\theta - \frac{\sin 2\theta}{2}\right] \times 2R = 0$$

$$2R^3\theta\left[1-\cos 2\theta\right] - 2R^3\left[\theta - \frac{\sin 2\theta}{2}\right] = 0$$

$$2R^3 = 0$$

$$\theta\left[1-\cos 2\theta\right] - \left[\theta - \frac{\sin 2\theta}{2}\right] = 0$$

$$2\theta = 257^\circ 30'$$

$$\boxed{\theta = 128^\circ 45'}$$

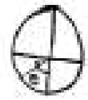
By trail and error method the solution of this equation is given by $128^\circ 45'$

From the figure, the depth of water is given by

$$d = OD - OC$$

$$\boxed{d = R - R\cos\theta}$$

$$d = R - R\cos 128^\circ 45'$$



$$d = R\,[1 - \cos\,[180 - 128^\circ 45')$$

$$d = R\,[1 - \cos\,(-51^\circ 15')]$$

$$d = R\,[1 + 0.62]$$

$$d = R \times 1.62$$

$$d = \frac{D}{2} \times 1.62$$

$$\boxed{d = D \times 0.81}$$

Hydraulic mean depth is given by

$$m = \frac{A}{P} = \frac{R^2\left[\theta - \frac{\sin 2\theta}{2}\right]}{2R\theta}$$

$$m = \frac{R}{2\theta}\left\{\theta - \frac{\sin 2\theta}{2}\right]$$

$$\theta = 128^\circ 45'$$

$$\theta = 128^\circ 45' \times \frac{\pi}{180}$$

$$\boxed{\theta = 2.247 \text{ rad}}$$

$$m = \frac{R}{2 \times 2.247} \left\{ 2.247 - \frac{\sin 2(2.247)}{2} \right\}$$

$$m = 0.611\, R$$

$$m = 0.611 \frac{D}{2}$$

$$\boxed{m = 0.3\, D}$$

condition for the maximum velocity is given by $\theta = 2.247$ radians

$$d = 0.81\, D$$

$$m = 0.3\, D$$

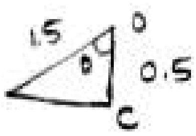
<u>condition for discharge</u>

$$\theta = 154^\circ$$

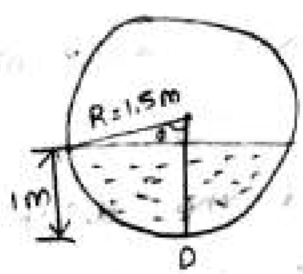
$$d = 0.95\, D$$

$$m = \frac{A}{P}$$

① Find the discharge through a circular pipe of diameter 3 m, if the depth of the water pipe is 1m and the pipe is laid at a slope of 1 in 1000. Take C = 70



$$OC = OD - CD$$
$$= 1.5 - 1$$
$$= 0.5$$



$$\cos\theta = \frac{0.5}{1.5}$$

$$\theta = \cos^{-1}\left[\frac{0.5}{1.5}\right] = 70.52^\circ$$

$$= 70.52 \times \left(\frac{\pi}{180}\right)$$

$$= 1.230 \text{ rad}$$

$$A = R^2\left[\theta^{\to \text{rad}} - \frac{\sin 2\theta^{\to \theta}}{2}\right]$$

$$= (1.5)^2\left[1.230 - \frac{\sin 2 \times 70.52}{2}\right]$$

$$\boxed{A = 2.06 \text{ m}^2}$$

$$P = 2R\theta^{\to \text{rad}}$$

$$= 2 \times 1.5 \times 1.230$$

$$\boxed{P = 3.69 \text{ m}}$$

$$m = \frac{A}{P} = \frac{2.06}{3.69}$$

$$\boxed{m = 0.558}$$

$$Q = AC\sqrt{mi}$$

$$= 2.06 \times 70 \sqrt{0.558 \times \frac{1}{1000}}$$

$$\boxed{Q = 3.4062 \text{ m}^3/\text{sec}}$$

2. Find the discharge through a circular pipe of diameter 3 m, if the depth of the

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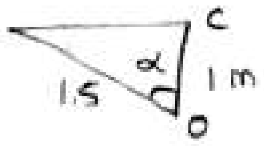
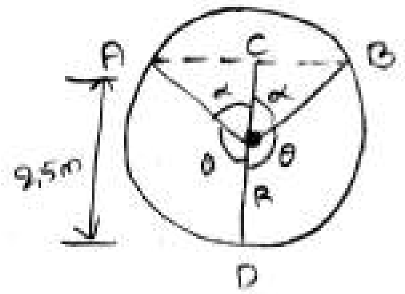
water pipe is 2.5 m and the pipe is laid at a slope of 1 in 1000. Take $C = 70$.





$OC = CD - OD$

$= 2.5 - 1.5$

$OC = 1\ m$

$\alpha = \cos^{-1}\left(\frac{1}{1.5}\right)$

$\boxed{\alpha = 48.18^\circ}$

$\theta = 180 - \alpha$

$\theta = 180 - 48.18$

$\theta = 131.82^\circ$

$= 2.30\ rad$

$A = R^2 \left[\underset{\to rad}{\theta} - \frac{\sin 2\underset{\to \theta}{\theta}}{2}\right]$

$= (1.5)^2 \left[2.30 - \frac{\sin 2 \times 131.82}{2}\right]$

$\boxed{A = 6.2930\ m^2}$

$P = 2R\theta$

$= 2 \times 1.5 \times 2.30$

$\boxed{P = 6.9}$

$m = \frac{A}{P} = \frac{6.29}{6.9} = 0.912\ m$

$Q = AC\sqrt{mi}$

$= 6.293 \times 70 \sqrt{0.912 \times \frac{1}{1000}}$

$\boxed{Q = 13.30\ m^3/sec}$

③ A concrete circular channel of diameter 3 m as a bed slope of 1 in 500 workout the velocity and flow rate for the condition

(1) maximum velocity $C = 50$

(2) maximum discharge

* maximum velocity

$\theta = 128^\circ 45'$

$= 2.247$ rad

$D = 3m$

$R = 1.5m$

$i = \frac{1}{500}$

$C = 50$

$A = R^2 \left[\theta - \frac{\sin 2\theta}{2}\right]$

$= (1.5)^2 \left\{2.247 - \frac{\sin 2 \times 128.45}{2}\right.$

$\boxed{A = 6.154\ m^2}$

$P = 2R\theta$

$= 2 \times 1.5 \times 2.247$

$\boxed{P = 6.741}$

$m = \frac{A}{P} = \frac{6.154}{6.741}$

$\boxed{m = 0.912}$

$$V = C\sqrt{mi}$$

$$= 50 \times \sqrt{0.912 \times \frac{1}{500}}$$

$$\boxed{V = 2.135 \ m/sec}$$

$$Q = A \times V$$

$$= 6.154 \times 2.135$$

$$\boxed{Q = 13.14 \ m^3/sec}$$

Maximum discharge :

$$\theta = 154^\circ$$

$$= 2.687 \ rad$$

$$A = R^2 \left[\theta - \frac{\sin 2\theta}{2}\right]$$

$$= (1.5)^2 \left[2.687 - \frac{\sin 2 \times 154^\circ}{2}\right]$$

$+0.394$

$$\boxed{A = 6.932 \ m^2}$$

$$P = 2R\theta$$

$$= 2 \times 1.5 \times 2.687$$

$$\boxed{P = 8.061}$$

$$m = \frac{A}{P} = \frac{6.932}{8.061}$$

$$\boxed{m = 0.859}$$

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$V = C\sqrt{mi}$

$= 50\sqrt{0.859 \times \frac{1}{500}}$

$$\boxed{V = 2.072 \text{ m/sec}}$$

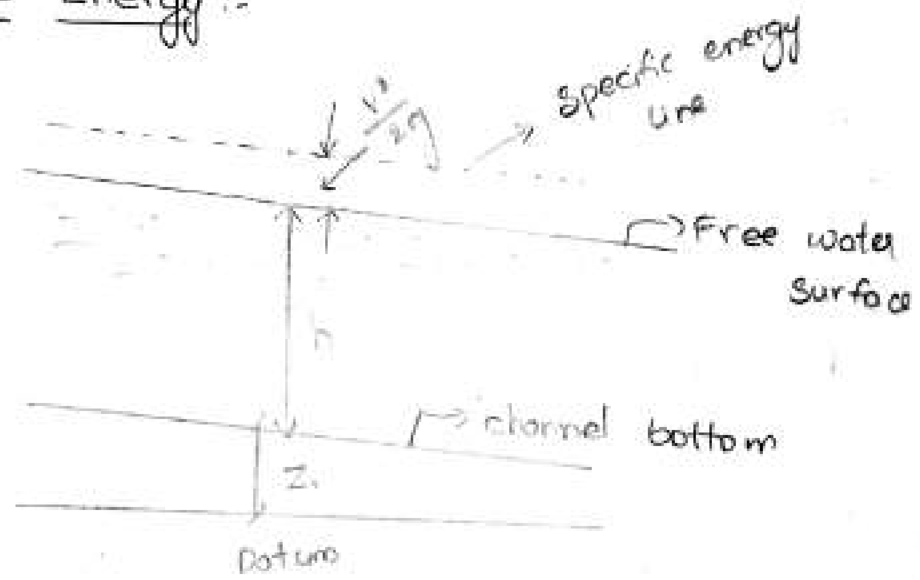
$Q = A \times V$

$= 6.932 \times 2.072$

$$\boxed{Q = 14.36 \text{ m}^3/\text{sec}}$$

Specific Energy and Specific Energy curve

Specific Energy :-



The total energy of flowing fluid per unit weight is given by,

$$\text{Total energy} = Z + h + \frac{V^2}{2g}$$

where, Z = height of the bottom of channel above datum

h = Depth of liquid

V = mean velocity of flow

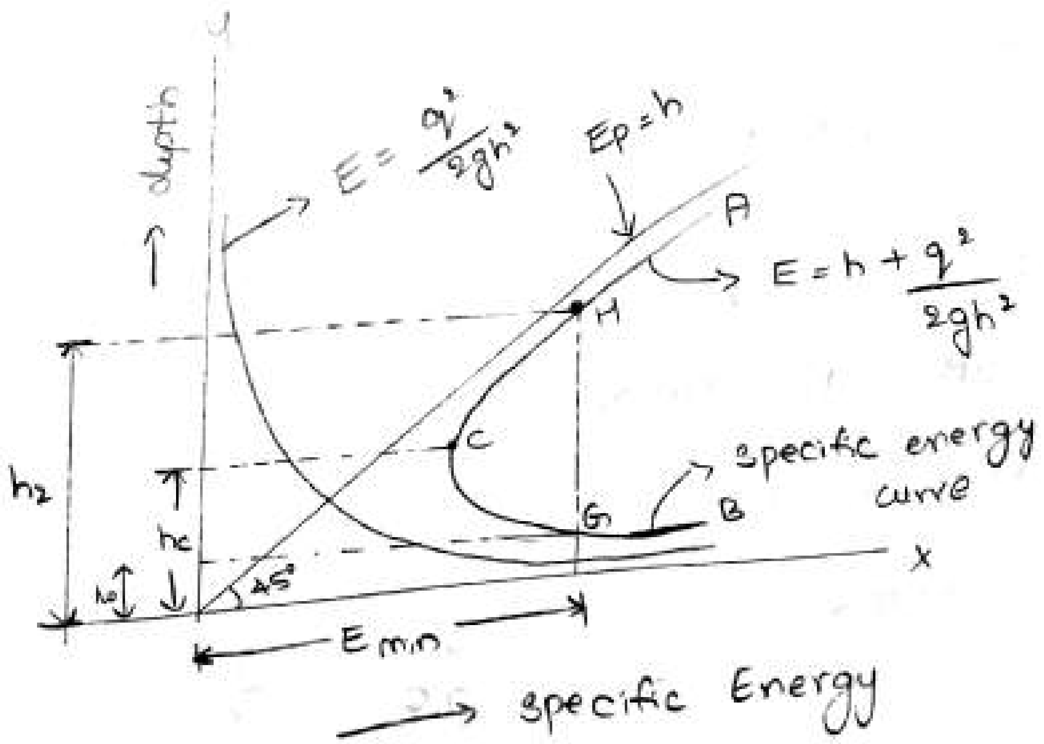
If the channel bottom is taken as the datum as shown in figure, then the total energy per unit weight of liquid will be,

$$\boxed{E = h + \frac{V^2}{2g}} \longrightarrow (1)$$

The energy given in the eqn (1) is known as specific energy hence the specific energy of the flowing fluid is defined as energy per unit weight of the liquid with respect to the bottom of the channel.

Imp ★

SPECIFIC ENERGY CURVE :-



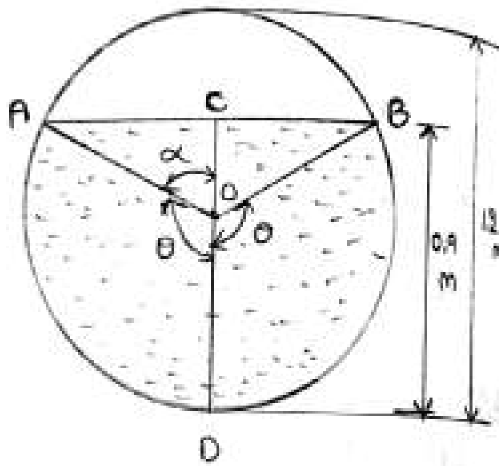
① Calculate the quantity of water that will be discharged at a uniform depth of 0.9 m in a 1.2 m diameter pipe which is laid at a slope 1 in 1000. Assume chezy's C = 58

Sol^n :- Given :-

Dia of pipe = 1.2 m

Radius $R = \frac{1.2}{6}$

$R = 0.6$ m



Depth of water, d = 0.9 m

Slope, $i = \frac{1}{1000}$

chezy's $C = 58$

From figure,

$OC = CD - OD$

$= 0.9 - R$

$= 0.9 - 0.6$

$\boxed{OC = 0.3 \text{ m}}$

$OA = R = 0.6$ m

From Δ^{le} AOC,

$$\cos\alpha = \frac{OC}{OA} = \frac{0.3}{0.6} = \frac{1}{2}$$

$$\alpha = \cos^{-1}\left(\frac{1}{2}\right) \qquad \boxed{\alpha = 60^\circ}$$

θ = angle DOA

$\theta = 180° - \alpha$

$\theta = 180° - 60°$

$\theta = 120°$

$\theta = 120° \times \frac{\pi}{180}$ radians

$\boxed{\theta = 2.094 \text{ rad}}$

$A = R^2 \left[\theta - \frac{\sin 2\theta}{2} \right]$

$= (0.6)^2 \left[2.094 - \frac{\sin 2 \times 120°}{2} \right]$

$\boxed{A = 0.91 \ m^2}$

$P = 2R\theta$

$= 2 \times 0.6 \times 2.094$

$\boxed{P = 2.5128}$

$m = \frac{A}{P} = \frac{0.91}{2.512}$

$\boxed{m = 0.362}$

$Q = AC\sqrt{mi}$

$= 0.91 \times 58 \times \sqrt{0.362 \times \frac{1}{1000}}$

$\boxed{Q = 1.004 \ m^3/s}$ 1.007

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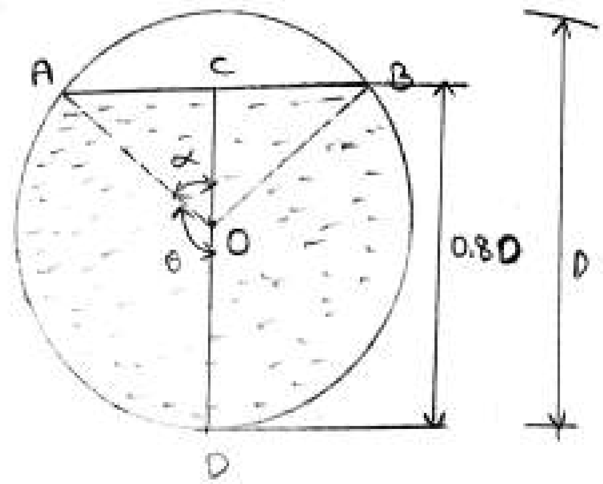
② water is flowing through a circular channel at the rate of 400 lit/s, when the channel is having a bed slope of 1 in 9000. The depth of water in the channel is 8.0 times the diameter. Find the diameter of the circular channel if the value of manning's N = 0.015.

Soln:



$Q = 400$ lit/s

$= 0.4\ m^3/s$

$i = \frac{1}{9000}$

$N = 0.015$

$d = 0.8D$

$D = ?$

From the figure,

$OC = CD - OD$

$= 0.8D - \frac{D}{2}$

$= (0.8 - 0.5)D$

$\boxed{OC = 0.3D}$

$AO = R$

$= \frac{D}{2}$

$= D \times \frac{1}{2}$

$\boxed{AO = 0.5D}$

$\cos\alpha = \frac{OC}{AO} = \frac{0.3D}{0.5D} = 0.6$

$$\cos\alpha = 0.6$$

$$\alpha = \cos^{-1} 0.6$$

$$\boxed{\alpha = 53.13^\circ}$$

$$\theta = 180^\circ - \alpha$$

$$= 180^\circ - 53.13$$

$$\theta = 126.87$$

$$\theta = 126.87 \times \frac{\pi}{180}$$

$$\boxed{\theta = 2.214 \text{ rad}}$$

$$P = 2R\theta$$

$$= 2 \times \frac{D}{2} \times 2.214$$

$$\boxed{P = 2.214\, D \text{ m}}$$

$$A = R^2 \left[\theta - \frac{\sin 2\theta}{2}\right]$$

$$= \left(\frac{D}{2}\right)^2 \left[2.214 - \frac{\sin(2 \times 126.87)}{2}\right]$$

$$= \frac{D^2}{4} \left[2.6940\right]$$

$$\boxed{A = 0.6735\, D^2}$$

$$m = \frac{A}{P} = \frac{0.6735\, D^{\cancel{2}}}{2.214\, \cancel{D}}$$

$$\boxed{m = 0.3042\, D}$$

manning's formula is given by

$$C = \frac{1}{N} m^{1/6}$$

$$c = \frac{1}{0.015}(0.3042\ D)^{1/6}$$

$$\boxed{D = 1.266\ m}$$

$$Q = AC\sqrt{mi}$$

$$0.4 = 0.6735\ D^2 \times \frac{1}{0.015}(0.3042\ D)^{1/6}$$

$$\sqrt{0.3042\ D \times \frac{1}{9000}}$$

$$0.4 = 0.6735 \times 54.672 \times 5.813 \times 10^{-3}$$

$$\left[D^2 \times D^{1/6} \times D^{1/2}\right]$$

$$D^{8/3} = 1.8685$$

$$\boxed{D = 1.264\ m}$$

(3) The rate of flow of water through a circular channel of diameter 0.6 m is 150 lit/s. Find the slope of the bed of the channel for maximum velocity. Take C = 60.

Sol Given :-

$D = 0.6$ m

$Q = 150$ lit/s

$Q = 0.15$ m^3/s

$C = 60$

$i = ?$

For the maximum velocity through a circular channel, depth of flow is given by

$$d = D \times 0.81$$

$$d = 0.6 \times 0.81$$

$$\boxed{d = 0.486 \text{ m}}$$

Condition for the maximum velocity is given by

$$\theta = 128^\circ 45'$$

$$= 128.45 \times \frac{\pi}{180}$$

$$\boxed{\theta = 2.247 \text{ rad}}$$

Hydraulic mean depth for maximum velocity is given by

$$m = 0.3 D$$

$$= 0.3 \times 0.6$$

$$\boxed{m = 0.18 \text{ m}}$$

perimeter for circular pipe

$P = 2R\theta$

$= D \times \theta$

$= 0.6 \times 2.247$

$\boxed{P = 1.3482 \text{ m}}$

$m = \frac{A}{P} =$

$A = m \times P$

$A = 0.18 \times 1.3482$

$\boxed{A = 0.2426 \text{ m}^2}$

$m = \frac{A}{P} = \frac{0.2426}{1.3482}$

$m = 0.179 \approx$

$\boxed{m = 0.18 \text{ m}}$

$\boxed{i = \frac{1}{1694.7}}$

$Q = AC\sqrt{mi}$

$0.15 = 0.2426 \times 60 \sqrt{0.18 \times i}$

$0.15 = 14.556 \sqrt{0.18 \times i}$

$0.0103 = \sqrt{0.18 \times i}$

$0.15 = 14.556 \times 0.4242 \times i^{1/2}$

$\frac{0.15}{6.1746} = i^{1/2}$

$i^{1/2} = 0.02429$

$i = (0.02429)^2$

$i = \frac{1}{5.9 \times 10^{-4}} = 1694.915$

$\boxed{i = 5.9 \times 10^{-4}}$

$\boxed{i = \frac{1}{694}}$

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(4) Determine the maximum discharge of water through a circular channel of diameter 1.5 m when the bed slope of the channel is 1 in 1000. Take C = 60.

Sol^n $D = 1.5$ m

$i = \frac{1}{1000}$

$C = 60$

$R = \frac{D}{2} = \frac{1.5}{2} = 0.75$ m

For maximum discharge.

$\theta = 154^\circ$

$\theta = 154^\circ \times \left(\frac{\pi}{180^\circ}\right)$

$\boxed{\theta = 2.6878 \text{ rad}}$

$P = 2R\theta$

$= 2 \times 0.75 \times 2.6878$

$\boxed{P = 4.0317 \text{ m}}$

$A = R^2 \left(\theta - \frac{\sin 2\theta}{2}\right)$

$= (0.75)^2 \left(2.6878 - \frac{\sin(2 \times 154^\circ)}{2}\right)$

$= (0.75)^2 \times 3.0818$

$\boxed{A = 1.7335 \text{ m}^2}$

$$m = \frac{A}{P} = \frac{1.7335}{4.0317}$$

$$\boxed{m = 0.4299}$$

maximum discharge

$$Q = AC\sqrt{mi}$$

$$= 1.7335 \times 60 \times \sqrt{0.4299 \times \frac{1}{1000}}$$

$$\boxed{Q = 2.1565 \ m^3/s}$$

SPECIFIC ENERGY CURVE

Specific Energy curve is defined as the curve which shows variation of specific energy of depth of flow and it is given by $E = h + \frac{v^2}{2g}$ = Potential Energy + K

Consider a rectangular channel in which a steady and non-uniform flow is taking place.

Let Q = Discharge of the channel

b = breadth of the channel

h = height of the channel

q = discharge per unit width of the channel.

$$q = \frac{Q}{b}$$

$$V = \frac{Q}{A}$$

$$V = \frac{Q}{b \times h}$$

$$V = \frac{q}{h}$$

$$SE = h + \frac{v^2}{2g} = E_p + E_k$$

$$\boxed{SE = h + \frac{q^2}{2gh^2}} \quad \text{—— (2)}$$

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Equation ② is known as Specific Energy with the depth of the flow hence for a given discharge 'Q' the different values of 'h' the corresponding 'E' is obtained.

Now a graph between Specific Energy (x-axis) along the depth of the flow [along Y-axis) is plotted.

Specific Energy curve diagram

The Specific Energy curve is obtained by first plotting the Specific energy.

i.e, $E_p = h$ which will be the straight line passing through origin making an $45°$ as shown in the figure.

* Then drawing the curve for kinetic energy $E_K = \frac{q^2}{2gh^2}$ which will be the parabola as shown in the figure

* By combining these two curves by we obtain specific energy curve.

* The curve ACB denotes the specific Energy as shown in the figure.

CRITICAL DEPTH :-

critical depth is defined as the depth of flow of water at which the specific energy is minimum. This is obtained by hc as shown in figure.

* From the figure the point C corresponds the minimum specific energy which is at the critical depth.

* The mathematical expression obtained by differentially the specific Energy equation with the depth of flow and equating to

$$\frac{dE}{dh} = 0$$

$$E = h + \frac{q^2}{2gh^2}$$

$$\frac{dE}{dh} = 1 + \frac{q^2}{2g}\left[\frac{-2}{h^3}\right] = 0$$

$$\frac{-\cancel{2}q^2}{\cancel{2}gh^3} = +1$$

$$q^2 = gh^3$$

$$gh^3 = q^2$$

$$h^3 = \frac{q^2}{g}$$

$$h = \sqrt[3]{\frac{q^2}{g}}$$

$$\boxed{h_c = \left[\frac{q^2}{g}\right]^{1/3}}$$

Critical velocity (V_c) :- The velocity of flow at the critical depth is known as critical velocity.

The expression for V_c is obtained by critical depth equation.

$$h^3 = \frac{q^2}{g}$$

$$h^3 \times g = q^2$$

$$h^3 \times g = \left[\frac{Q}{b}\right]^2$$

$$h^3 \times g = \left[\frac{A \times v}{b}\right]^2$$

$$h^3 \times g = \left[\frac{\cancel{b} \times h \times v}{\cancel{b}}\right]^2$$

$$h^{\cancel{3}} \times g = \cancel{h^2} \times v^2$$

$$h \times g = v^2$$

$$\boxed{v_c = \sqrt{h_c \times g}}$$

Minimum Specific Energy in terms of critical depth

$$E_{min} = h_c + \frac{q^2}{2gh_c^2}$$

$$E_{min} = \left(\frac{q^2}{g}\right)^{1/3} + \frac{q^2}{2g \times \left[\left(\frac{q^2}{g}\right)^{1/3}\right]^2}$$

$$\boxed{E_{min} \quad \frac{3h_c}{2}}$$

problems

① Find the specific energy of the flowing fluid through a rectangular channel of width 5 m and dischange is 10 m^3/s and the depth of water is 3m.

Soln

$b = 5 m$

$Q = 10 m^3/s$

$d = 3 m$

$A = b \times d$

$= 5 \times 3$

$\boxed{A = 15 m^2}$

$V = \frac{Q}{A}$

$= \frac{10}{15}$

$\boxed{V = 0.66 m/s}$

$q = \frac{Q}{b}$

$= \frac{10}{5}$

$\boxed{q = 2}$

$SE = h + \frac{q^2}{2gh^2}$

$SR = h + \frac{v^2}{2g}$

$= 3 + \frac{(0.66)^2}{2 \times 9.81}$

$\boxed{SR = 3.022 m}$

$SE = h + \frac{q^2}{2gh^2}$

$= 3 + \frac{(2)^2}{2 \times 9.81 \times 3^2}$

$\boxed{SE = 3.022 m}$

(2) Find the critical depth and critical velocity of water flowing through a rectangul channel of width 5 m and the discharge is 15 m³/s

Solⁿ critical depth

$$h_c = \left[\frac{q^2}{g}\right]^{1/3}$$

$$q = \frac{Q}{b} = \frac{15}{5}$$

$$\boxed{q = 3}$$

$$= \left[\frac{(3)^2}{9.81}\right]^{1/3}$$

$$\boxed{h_c = 0.9716}$$

critical velocity, $V_c = \sqrt{g \times h_c}$

$$= \sqrt{9.81 \times 0.9716}$$

$$\boxed{V_c = 3.0872 \text{ m/s}}$$

(3) The discharge of water through a rectangular channel of width 8 m is 15 m³/s. when the depth of flow of water is 1.2 m. calculate

(i) Specific Energy of the flowing water

(ii) critical depth and critical velocity

(iii) Value of minimum Specific Energy

Solⁿ

$b = 8$ m

$Q = 15$ m³/s

$d = 1.2$ m

$A = b \times d$

$A = 8 \times 1.2$

$\boxed{A = 9.6 \text{ m}^2}$

(i)

$$SE = h + \frac{q^2}{2gh^2}$$

$$= 1.2 + \frac{(1.875)^2}{2 \times 9.81 \times (1.2)^2}$$

$$\boxed{SE = 1.324 \text{ m}}$$

$$q = \frac{Q}{b}$$

$$= \frac{15}{8}$$

$$\boxed{q = 1.875}$$

$$SE = h + \frac{V^2}{2g}$$

$$= 1.2 \times \frac{(1.5625)^2}{2 \times 9.81}$$

$$\boxed{SE = 1.324 \text{ m}}$$

$$V = \frac{Q}{A} = \frac{15}{9.6}$$

$$\boxed{V = 1.5625 \text{ m/s}}$$

(ii)

$$h_c = \left[\frac{q^2}{g}\right]^{1/3}$$

$$= \left[\frac{(1.875)^2}{9.81}\right]^{1/3}$$

$$\boxed{h_c = 0.7103}$$

$$V_c = \sqrt{g \times h_c}$$

$$= \sqrt{9.81 \times 0.7103}$$

$$\boxed{V_c = 2.639 \text{ m/s}}$$

(iii)

$$E_{min} = \frac{3h_c}{2}$$

$$= \frac{3 \times 0.7103}{2}$$

$$\boxed{E_{min} = 1.0654}$$

Critical flow :-

It is defined as the flow at which Specific ENErgy is minimum or the flow corresponding to critical depth and is given by

$$V_c = \sqrt{g \times h_c}$$

$$h_c = \frac{V_c^2}{g}$$

$$\boxed{\frac{V_c}{\sqrt{g \times h_c}} = 1}$$

Froude's number =1

when froude's number is equal to 1 then is said to be critical flow.

Sub critical flow or streaming flow :-

when the depth of flow in the channel is greater than the critical depth then the flow is said to be Sub critical flow and also for this type of flow the froude's number should be less than 1.

$$h > h_c$$

$$F_e < 1 \rightarrow \text{Subcritical flow.}$$

Super critical flow or shooting flow or torrential flow

When the depth of flow in the channel is less than the critical depth then the flow is said to be Supercritical flow.

$$h < h_c$$

$$F_e > 1 \longrightarrow \text{supercritical flow}$$

Condition for the maximum discharge for the given value of maximum specific energy.

$$E = \frac{3h}{2}$$

16.36
16.37

or

$$h = \frac{2}{3} E$$

UNIFORM FLOW IN OPEN CHANNELS.

Introduction : Flow in open channels is defined as the flow of a liquid with a free surface Thus a liquid flowing at atmospheric pressure through a passage is known as flow in open channels The flow of water through pipes at atmospheric pressure or when the level of water in the pipe is below the top of the pipe is too considered as open channel flow

Classification of flow in channels

i) Steady flow and unsteady flow : If the flow characteristics such as depth of flow, velocity of flow, rate of flow at any point in open channel flow do not change with respect to time. the flow is said to be steady flow

$$\frac{\partial y}{\partial t} = 0 , \frac{\partial Q}{\partial t} = 0 , \frac{\partial V}{\partial t} = 0$$

If at any point in open channel flow, the velocity, depth and rate of flow changes with respect to time, the flow is said to be unsteady flow

$$\frac{\partial y}{\partial t} \neq 0 , \frac{\partial Q}{\partial t} \neq 0 , \frac{\partial V}{\partial t} \neq 0$$

ii) Uniform flow and Non uniform flow : If the flow characteristics such as depth of flow, velocity of flow, rate of flow at any point in open channel flow do not change with respect to space is said to be uniform flow

$$\frac{\partial y}{\partial s} = 0 , \frac{\partial V}{\partial s} = 0 , \frac{\partial Q}{\partial s} = 0$$

If at any point in open channel flow. the flow characteristics change with respect to space is said to be non-uniform flow

$$\frac{\partial y}{\partial s} \neq 0 , \frac{\partial V}{\partial s} \neq 0$$

Non uniform flow is also called as Varied flow which is classified as

(i) Rapidly Varied flow

(ii) Gradually Varied flow

Rapidly Varied flow : It is defined as that flow in which the depth of flow changes abruptly over a small length of the channel

Gradually Varied flow : If the depth of flow in a channel changes gradually over a long length of the channel, the flow is said to be gradually varied flow

iii) Laminar flow and Turbulent flow : The flow in open channel is said to be laminar if the Reynolds number is less than 600 $Re = \frac{\rho v d}{\mu}$

v - velocity

d - hydraulic mean depth

μ - viscosity

If the Reynolds number is more than 2000, the flow is said to be turbulent in open channel flow

If Re lies b/n 600 and 2000, the flow is considered to be in transition state

iv) Sub critical, Critical and Super Critical flow : If the Froude number (Fe) is less than 1, then the flow in open channel is said to be subcritical. Also called as tranquil flow. The flow is critical if Fe = 1, the flow is super critical if Fe is greater than 1, also called Rapid flow or torrential flow

$$F_e = \frac{V}{\sqrt{gD}}$$

Flow in Open channels

Comparison b/n Pipe flow and channel flow

Pipe flow

* In pipe flow, the flow of water takes place with no free surface
* The water level in the piezometric tube installed at different sections of a pipe indicate the hydraulic gradient
* Due to difference in pressure flow takes place in pipe flow
* The water flowing in pipe is always under pressure

Channel flow

* In channel flow flow takes place with free surface which is subjected to atmospheric pressure
* The water surface itself is the hydraulic gradient
* The weight of flowing water towards the direction of flow with a sloping bottom causes flow of water in a channel
* The water flowing is with free surface in atmospheric pressure

Open channel and closed channel

The channels without any cover at the top are known as Open channel

The channels with cover at the top is called closed channel and the flow in closed channel is always partial

Prismatic and Non Prismatic channel

A channel having the same shape of various sections along its length and laid on a constant bottom slope is known as Prismatic channel, otherwise the channel is Non Prismatic

Types of flow

(i) Steady and Unsteady flow.

Flow in a channel is said to be steady if the flow characteristics at any point donot change with time, otherwise the flow is unsteady.

2) Uniform and Non-uniform flow (Varied flow)

Flow in a channel is said to be uniform if the depth, slope, c/s and velocity remain constant over a given length of the channel.

Flow in channel is termed as non uniform or varied if the depth of flow changes from section to section along the length of the channel.

→ Non uniform

(a) Rapidly Varied flow : If the depth of flow changes abruptly over a short distance, then it is Rapidly Varied flow.

Ex :- Hydraulic jump and hydraulic drop.

(b) Gradually Varied flow : If the change in depth of flow takes place gradually in a long reach of the channel, then it is Gradually Varied flow.

3) Laminar and Turbulent flow

The flow is characterized as laminar, turbulent or in a transitional state, depending on the relative effect of viscous and inertia forces and Reynold's number Re is a measure of this effect in channel flow.

Reynold's number $Re = \frac{\rho v R}{\mu}$

$v \rightarrow$ mean velocity
$R \rightarrow$ hydraulic radius
$\rho \rightarrow$ mass density
$\mu \rightarrow$ absolute viscosity

Re equal to 500 to 600, the flow is laminar.

Re b/n 500 to 2000, the flow is transitional

Re greater than 2000, the flow is turbulent

4) Subcritical flow, Critical flow and Supercritical flow.

Gravity is a predominant force in case of channel flow. The ratio of inertia and the gravity force is another parameter called Froude's number Fr and.

$$Fr = \frac{v}{\sqrt{gD}}$$

$v \rightarrow$ mean velocity of flow.
$g \rightarrow$ accln due to gravity
$D \rightarrow$ hydraulic depth.

Fr = 1, the flow is in critical state

Fr < 1, the flow is in subcritical / tranquil / streaming

Fr > 1, the flow is in supercritical / rapid / torrential.

Geometrical Properties of Channel Section

a) Depth of flow (y) : It is the vertical distance of the lowest point of a channel section from the free surface

b) Top width (T) : It is the width of the channel section at the ~~direction~~ ~~of~~ ~~flow~~ free surface

c) Wetted Area (A): It is the c/s area of the flow normal to the direction of flow

d) Wetted Perimeter (P): It is the length of channel boundary in contact with the flowing water at any section

e) Hydraulic Radius (R): It is the ratio of the wetted area to its wetted perimeter

$$R = A/P$$

f) Hydraulic Depth (D): It is the ratio of the wetted area to the top width.

$$D = A/T$$

g) Section factor (Z): For critical flow, Z is the product of the wetted area and the square root of the hydraulic depth.

$$Z = A\sqrt{D} = \left[\frac{A^3}{T}\right]^{1/2}.$$

for uniform flow,

$$Z = AR^{2/3}.$$

Geometric elements of channel sections

i) Rectangle

Area (A) = By

Wetted Perimeter (P) = $B+2y$.

Hydraulic Radius (R) = $\frac{By}{B+2y}$.

Top width (T) = B

Hydraulic depth (D) = y

Section factor (Z) = $By^{3/2}$

(2) Trapezoidal

$$\text{Area }(A) = (B+zy)y$$

$$P = B + 2y\sqrt{z^2+1}$$

$$R = \frac{(B+zy)y}{B+2y\sqrt{z^2+1}}$$

$$T = B + 2zy$$

$$D = \frac{(B+zy)y}{B+2zy}$$

$$Z = \frac{[(B+zy)y]^{3/2}}{(B+2zy)^{1/2}}$$

(3) Triangle

$$A = zy^2$$

$$P = 2y\sqrt{z^2+1}$$

$$R = \frac{zy}{2\sqrt{z^2+1}}$$

$$T = 2zy$$

$$D = \frac{1}{2}y$$

$$Z = \frac{zy^{3/2}}{\sqrt{2}}$$

(4) Circle

$$A = \frac{1}{8}(\theta - \sin\theta)D^2$$

$$P = \frac{1}{2}\theta D$$

$$R = \frac{1}{4}\left[1 - \frac{\sin\theta}{\theta}\right]D$$

$$T = 2\sqrt{y(D-y)} \quad \text{or } D\sin\theta/2$$

$$D = \frac{1}{8}\left(\frac{\theta - \sin\theta}{\sin\theta/2}\right)D$$

(5) Parabola

$$A = \frac{2}{3}Ty$$

$$P = T + \frac{8}{3}\frac{y^2}{T}$$

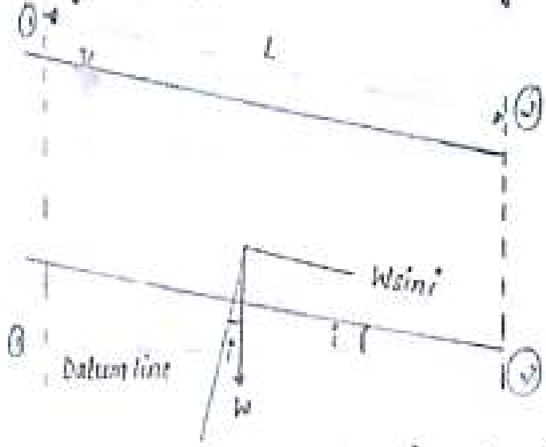
$$R = \frac{2T^2y}{3T^2+8y^2}$$

$$T = \frac{3A}{2y}$$

$$D = \frac{2}{3}y$$

$$Z = \frac{2}{9}\sqrt{6}$$

Discharge through Open channel by Chezy's formula



Consider uniform flow of water in a chanel as shown. As the flow is uniform, flow properties will be constant for a given length of the channel.

Let L = length of channel.
A = area of flow of water.
i = slope of the bed
V = mean velocity of flow of water
P = wetted perimeter of the cross section.
f = frictional resistance per unit velocity per unit area.

Weight of water between sections (1) – (1) & (2) – (2).

W = Specific weight of water × volume of water.
$= w \times A \times L$

Component of weight of water along the direction of flow $= W \sin i = w \times A \times L \times \sin i$

Frictional resistance against motion of water $= f \times$ surface area $\times (\text{velocity})^n$

By experiment $n = 2$.

$\therefore$ Frictional resistance $= f \times P \times L \times V^2$

The forces acting on the water b/n the 2 sections

i) Component of weight of water along the direction of flow
ii) Frictional resistance against the flow
iii) Pressure force at section (1) – (1)
iv) Pressure force at section (2) – (2)

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Depth of water at section (1) and (2) are same, the pressure forces are same at these 2 sections and since they act opposite to each other, they cancel each other.

Resolving the forces along the direction of flow

$$wAL\sin i - f \times P \times L \times V^2 = 0$$

$$V^2 = \frac{wAL\sin i}{f \times P \times L}$$

$$V = \sqrt{\frac{w}{f}} \times \sqrt{\frac{A}{P} \times \sin i}$$

$\frac{A}{P}$ = hydraulic radius or hydraulic mean depth

$\frac{A}{P}$ = 'm'

$\sqrt{\frac{w}{f}}$ = C = Chezy's constant

$\sin i \simeq i$

$$\therefore \boxed{V = C\sqrt{mi}}$$

Conveyance of the section

$$\boxed{k = AC\sqrt{m}}$$

Discharge = A × V

$$\boxed{Q = A \times C\sqrt{mi}}$$

Problem.

(1) Find the velocity of flow and rate of flow of water through a rectangular channel of 6m wide and 3m deep, when it is running full. The channel is having bed slope as 1 in 2000. Take C = 55.

Soln:

B = 6m

d = 3m

i = 1/2000

C = 55

V = ?

Q = ?

By Chezy's formula

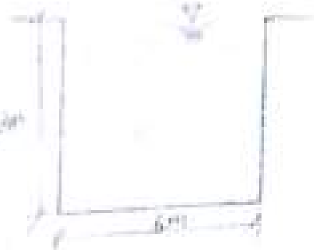
$$V = C\sqrt{mi}$$

$$V = 55\sqrt{\frac{A}{P} \times \frac{1}{2000}}$$

$A = B \times d = 6 \times 3 = 18 \, m^2$

$P = B + 2d = 6 + 2(3) = 12 \, m$

$m = \frac{A}{P} = \frac{18}{12} = 1.5 \, m$



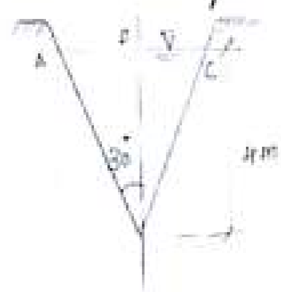
$V = 55 \sqrt{\frac{1.5 \times 1}{2000}}$

$V = \underline{1.506 \, m/s}$

$Q = V \times Area = 1.506 \times 18$

$= \underline{27.108 \, m^3/s}$

2) Find the rate of flow through a V shaped channel as shown if the value of $C = 55$ and bed slope is 1 in 2000.



Soln $i = \frac{1}{2000}$

$C = 55$

$A = \frac{1}{2} \times 2 \times AD \times 4$

$= 9.237 \, m^2$

$\tan 30° = \frac{AD}{DB}$

$AD = 4 \times \tan 30°$

$\cos 30° = \frac{BD}{AB}$

$AB = \frac{BD}{\cos 30°}$

$AB = \frac{4}{\cos 30°}$

$P = 2 \times AB$

$= \frac{2 \times 4}{\cos 30°}$

$= 9.2375 \, m$

$m = \frac{A}{P} = \frac{9.237}{9.2375}$

$m = 1.0 \, m$

$V = C\sqrt{m \, i}$

$= 55 \sqrt{1 \times \frac{1}{2000}}$

$= \underline{1.22 \, m/s}$

$Q = A \times V$

$= 9.237 \times 1.22$

$= 11.226 \, m^3/s$

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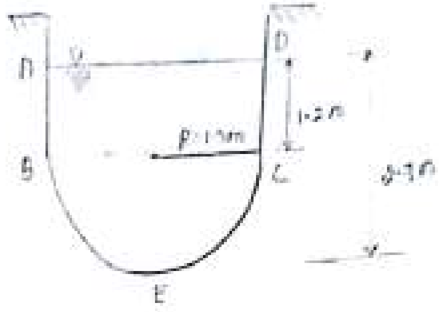
3) Find the discharge of water through the channel shown in fig. Take $C = 60$ and slope of the bed as 1 in 2000.

Soln:



$C = 60$

$i = 1/2000$

$$A = 3 \times 1.2 + \frac{\pi R^2}{2}$$

$$= 3 \times 1.2 + \left(\frac{\pi \times 1.5^2}{2}\right)$$

$$= 7.134\ m^2$$

$$P = 1.2 + 1.2 + \frac{2\pi R}{2}$$

$$= 1.2 + 1.2 + (\pi \times 1.5)$$

$$= 7.114\ m$$

$$m = \frac{A}{P} = 1.003\ m$$

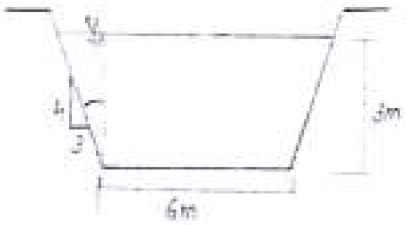
$$Q = AC\sqrt{mi}$$

$$= 7.134 \times 60 \times \sqrt{1.003 \times \frac{1}{2000}}$$

$$= 9.585\ m^3/s$$

4) Find the bed slope of trapezoidal channel of bed width 6m, depth of water 3m and side slope of 3 horizontal to 4 vertical, when the discharge through the channel is $30\ m^3/s$. Take $C = 70$. Also find the conveyance.

Soln:

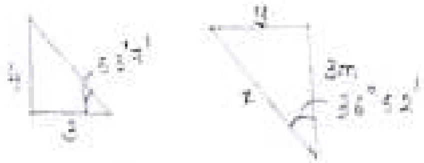


$b = 6m$

$d = 3m$

$$A = (6 \times 3) + 2 \times \left[\frac{1}{2} \times 3 \times 2.25\right]$$

$$= 24.75\ m^2$$



$$P = 2 \times 3.75 + 6$$

$$= 13.5\ m$$

$$m = 24.75/13.5 = 1.84\ m$$

$$Q = AC\sqrt{mi}$$

$$\frac{30}{24.75 \times 70 \times \sqrt{1.84}} = i^{1/2}$$

$\cos 36°52' = \frac{3}{z}$

$z = 3.75m$

$y = 3.75 \times \sin 36°52'$

$y = 2.25m$

$$i = \frac{1}{\left(\frac{2359}{50}\right)^2}$$

$$\boxed{i = \frac{1}{6136}}$$

$$k = AC\sqrt{m}$$

$$= 24.75 \times 70\sqrt{1.34}$$

$$k = \underline{\underline{2359}}$$

MANNING'S FORMULA

$$C = \frac{1}{N} m^{1/6}$$

$$\boxed{Q = A \frac{1}{N} m^{1/6} \sqrt{mi}}$$

N = Manning's constant

5) Find the discharge through a rectangular chanel of width 2 m, having a bed slope of 4 in 8000. The depth of flow is 1.5 m and take the value of N in Manning's formula as 0.012.

Soln. $b = 2m$

$i = 4/8000$

$d = 1.5m$

$N = 0.012$

$$C = \frac{1}{N} m^{1/6}$$

$$= \frac{1}{0.012} \left(\frac{2 \times 1.5}{2+1.5+1.5}\right)$$

$$C = 76.54$$

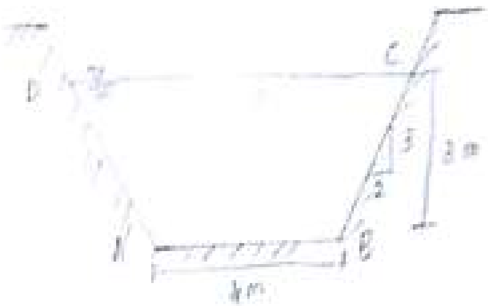
$$Q = AC\sqrt{mi}$$

$$= 2 \times 1.5 \times 76.54 \times \sqrt{0.6 \times \frac{4}{8000}}$$

$$= \underline{3.977} \ m^3/s.$$

6) Find the bed slope of trapezoidal channel of bed width 4m, depth of water 3m and side slope of 2H to 3V, when the discharge through the channel is 20m³/s Take N = 0.03 in Manning's formula

Soln



$i = ?$

$b = 4m$

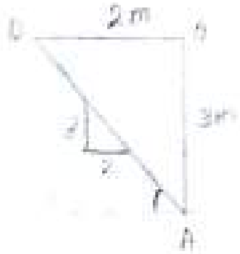
$d = 3m$

$Q = 20 m^3/s$

$N = 0.03$

$$A = (4 \times 3) + 2\left[\frac{1}{2} \times 2 \times 3\right]$$

$$= 18 m^2$$



$$\theta = \tan^{-1}\left(\frac{3}{2}\right) = 56°18'$$

$$DA = \sqrt{2^2 + 3^2} = 3.6 m$$

$$P = 3.6 + 4 + 3.6 = 11.2 m.$$

$$m = \frac{A}{P} = \frac{18}{11.2} = 1.6$$

$$C = \frac{1}{N} m^{1/6} = \frac{1}{0.03} \times (1.6)^{1/6}$$

$$= 36.07$$

$$Q = 18 \times 36.07 \sqrt{1.6 \times i}$$

$$\frac{20}{(18 \times 36.07 \times 1.6^{1/2})} = i^{1/2}$$

$$\boxed{i = \frac{1}{1692}}$$

7) Find the diameter of a circular sewer pipe which is laid at a slope of 1 in 8000 and carries a discharge of 800 lps when flowing half full. Take the value of Manning's N = 0.020.

Soln

$D = ?$

$i = 1/8000$

$Q = 800 \times 10^{-3} m^3/s$

$N = 0.020$

$$C = \frac{1}{N} m^{1/6}$$

$$C = \frac{1}{0.020}\left[\frac{A}{P}\right]^{1/6}$$

$$A = \frac{\pi\left[\frac{D}{2}\right]^2}{2} = \frac{\pi D^2}{8}$$

$$P = \frac{\pi D}{2}$$

$$m = \frac{A}{P} = \frac{\pi D^2}{8} \times \frac{2}{\pi D}$$

$$m = \frac{D}{4}$$

$$C = \frac{1}{N} m^{1/6}$$

$$C = \frac{1}{0.020}\left(\frac{D}{4}\right)^{1/6}$$

$$Q = A C \sqrt{m i}$$

$$0.8 = \frac{\pi D^2}{8} \times 0.1/0.020 \times \frac{D^{1/6}}{4^{1/6}} \sqrt{\frac{D}{4} \times \frac{1}{8000}}$$

$$\frac{0.8 \times 8 \times 0.020 \times 4^{1/6}}{\pi \times 1 \times \sqrt{\frac{1}{4 \times 8000}}} = D^{1/2} D^{4/1} D^{1/6}$$

$$9.1849 = D^{8/3}$$

$$D = \underline{2.296\ m}$$

Most Economical Section of Channels

A section of a channel is said to be most economical when the cost of construction of the channel is minimum. But the cost of construction depends on excavation and lining. To keep the cost minimum, the wetted perimeter for a given ~~cross sectional area (A)~~ discharge should be minimum.

Discharge Q will be maximum, when wetted Perimeter P is minimum

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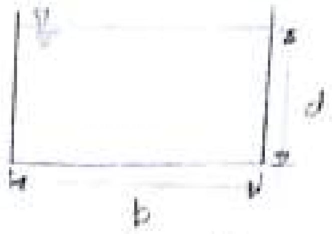
Most economical Rectangular Channel

The condition for most economical section, is that for a given area, the perimeter should be minimum



Consider a rectangular channel as shown in fig.

Area of flow $= b \times d \longrightarrow b = A/d$ ——— ①

Wetted Perimeter $= b + 2d$

$$P = \frac{A}{d} + 2d \quad \text{———} \quad ②$$

For most economical section, P should be min

$$\frac{d(P)}{d(d)} = 0$$

Differentiating the eqn ② w.r.to d.

$$\frac{d}{d(d)}\left[\frac{A}{d} + 2d\right] = 0$$

$$-\frac{A}{d^2} + 2 = 0$$

$$A = 2d^2 \quad \text{———} \quad ③$$

But $A = b \times d$

$$b \times \cancel{d} = 2d^{\cancel{2}}$$

$$\boxed{b = 2d}$$

$$m = \frac{A}{P} = \frac{b \times d}{b + 2d} = \frac{2d \times d}{2d + 2d} = \frac{\cancel{2}d^{\cancel{2}}}{\cancel{4}\cancel{d}_{2}}$$

$$\boxed{m = \frac{d}{2}}$$

∴ A rectangular channel will be most economical when (i) $b = 2d$, i.e width is two times of depth.

(ii) $m = \frac{d}{2}$ i.e hydraulic depth is half the depth of flow

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A rectangular channel 4 m wide has depth of water 1.5 m. The slope of the bed of the channel is 1 in 1000 and C = 55. It is desired to increase the discharge to a max by changing the dimensions of the section for constant area of c/s, slope of the bed and roughness of the channel. Find the new dimensions and increase in discharge.

Sol:n $b = 4m$

$d = 1.5m$

$i = 1/1000$

$C = 55$

$A = b \times d$

$= 4 \times 1.5$

$= 6\ m^2$

$P = b + 2d = 4 + 3 = 7m$

$m = \frac{A}{P} = \frac{6}{7} = 0.857$

$Q = AC\sqrt{mi} = 6 \times 55 \sqrt{0.857 \times \frac{1}{1000}}$

$= 9.66\ m^3/s$

For max discharge,

Conditions for economical section

(1) $b = 2d$

(2) $m = d/2$

Let the new dimensions be b' and d'

$A = b' \times d'$

Since c/s area is constant

$6 = b' \times d'$

$6 = 2d' \times d'$

$3 = d'^2$

$d' = 1.73\ m$

$b' = 2d'$

$= 2 \times 1.73$

$= 3.464\ m$

$P = 3.464 + 1.73 + 1.73 = 6.93\ m$

$$m = \frac{A}{P} = \frac{6}{6.93} = 0.866$$

$$Q = AC\sqrt{mi}$$

$$= 6 \times 55 \sqrt{0.866 \times \frac{1}{9000}}$$

$$= 9.71 \ m^3/s$$

Increase in discharge $= 9.71 - 9.66$

$= 0.05 \ m^3/s$

1) A rectangular channel of width 4m is having a bed slope of 1 in 1500. Find the max discharge through the channel. Take value of $C = 50$.

Soln: $i = 1/1500$

$C = 50$.

$b = 4m$

$Q_{max} = ?$

Conditions for max discharge

(i) $b = 2d$

(ii) $m = d/2$

$4 = 2d$

$d = 4/2 = 2m$

$m = \frac{2}{2} = 1m$

$$Q_{max} = AC\sqrt{mi}$$

$$= (4\times2)\times 50\sqrt{1\times\frac{1}{1500}}$$

$$= 10.328\ m^3/s.$$

2) A rectangular channel carries water at the rate of 400 lps when bed slope is 1 in 2000. Find the most economical dimensions of the channel if $C = 50$.

Soln: $Q = 400\times10^{-3}\ m^3/s$

$i = 1/2000$

$C = 50$

Conditions for max discharge.

$b = 2d$

$m = d/2$

$$Q = AC\sqrt{mi}$$

$A = b\times d$

$= 2d\times d$

$= 2d^2$

$$0.4 = 2d^2\times 50\sqrt{\frac{d}{2}\times\frac{1}{2000}}.$$

$$\frac{0.4\times\sqrt{2000\times2}}{50\times2} = d^2\cdot d^{1/2}.$$

$$d^{5/2} = 0.253$$

$d = 0.577m$

$b = 2d = 2\times0.577$

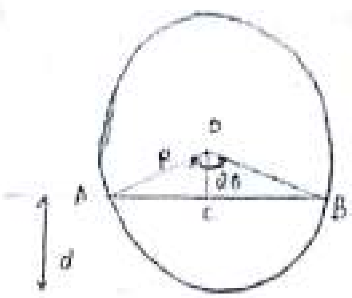
$= 1.154m$

Flow through Circular Section.

Let d = depth of water.

2θ = angle subtended by water surface AB at the centre in radians

R = radius of the channel.



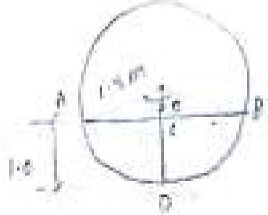
$$P = 2R\theta$$

$$A = R^2\left[\theta - \frac{\sin 2\theta}{2}\right]$$

$$m = \frac{R}{2\theta}\left[\theta - \frac{\sin 2\theta}{2}\right]$$

10) Find the discharge through a circular pipe of dia 3 m if the depth of water in the pipe is 1.0 m and the pipe is laid at a slope of 1 in 1000. Take $C = 70$.

Soln:



$D = 3m$

$R = 1.5m$

$d = 1m$

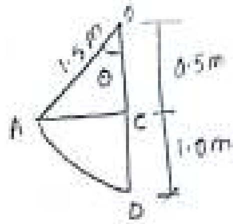
$i = \frac{1}{1000}$

$C = 70$.

$P = 2R\theta$

$= 2\times 1.5\times 1.23$.

$P = 3.7\ m$



$$A = R^2\left[\theta - \frac{\sin 2\theta}{2}\right]$$

$$= 1.5^2\left[1.23 - \frac{\sin 2(70^\circ 31')}{2}\right]$$

$$= 2.07\ m^2.$$

In ΔOCA

$\cos\theta = \frac{0.5}{1.5}$

$\theta = 70^\circ 31'$.

$70.53\times\frac{\pi}{180} = 1.23$ in radians.

$$m = \frac{A}{P} = \frac{2.07}{3.7} = 0.558.$$

$$Q = AC\sqrt{mi}$$

$$= 2.07\times 70\sqrt{0.56\times\frac{1}{1000}}$$

$$= 3.407\ m^3/s.$$

11) Find the discharge through a circular pipe of dia 3m, if the depth of water is 2.5m. find the rate of flow through it and the pipe is laid at a slope of 1 in 1000 Take C = 70.

Soln

$Q = ?$

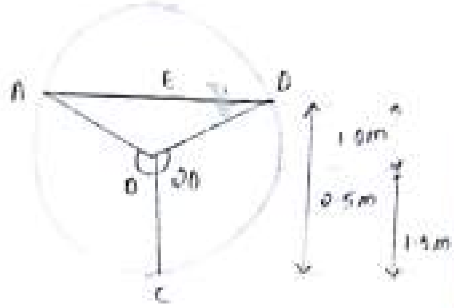
$D = 3m$

$R = 1.5m$

$d = 2.5m$

$i = \frac{1}{1000}$

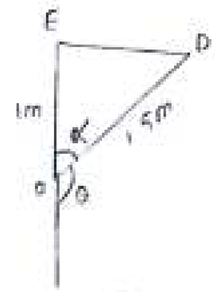
$C = 70$



$P = 2R\theta$

$= 2 \times 1.5 \times 2.3$

$P = 6.9 \ m$



$\cos\alpha = \frac{1}{1.5}$

$\alpha = 48^\circ 11'$

$\theta = 131^\circ 48'$

$\theta = 2.3$ radians

$$A = R^2\left[\theta - \frac{\sin 2\theta}{2}\right]$$

$$= 1.5^2\left[2.3 - \frac{\sin 2(131^\circ 48')}{2}\right]$$

$$A = 6.29 \ m^2$$

$$m = \frac{A}{P} = 0.91 \ m$$

$$Q = AC\sqrt{mi}$$

$$= 6.29 \times 70 \times \sqrt{0.91 \times \frac{1}{1000}}$$

$$= \underline{13.303} \ m^3/s$$

12) Water is flowing through a circular channel at the rate of 400lps, when the channel is having a bed slope of 1 in 9000. The depth of water in the channel is 0.8 times the dia. Find the dia of the channel if N = 0.015

Most Economical Circular Section.

In circular section there are two conditions for economic section <1> Condition for max velocity.
<2> Condition for max discharge

Condition for max velocity.

Let d = depth of flow of water

2θ : angle subtended at the centre by water surface

R = radius of channel

i = slope of the bed.

$$V = AC\sqrt{mi}$$

$$= AC\sqrt{\frac{A}{P}i}$$

$$\frac{d\left(\frac{A}{P}\right)}{d\theta} = 0$$

$$\frac{P\cdot\frac{dA}{d\theta} - A\frac{dP}{d\theta}}{P^2} = 0 \qquad \text{——— (1)}$$

$$A = R^2\left[\theta - \frac{\sin 2\theta}{2}\right]$$

$$\frac{dA}{d\theta} = R^2\left[1 - \frac{\cos 2\theta \times 2}{2}\right]$$

$$= R^2(1 - \cos 2\theta)$$

$$P = 2R\theta$$

$$\frac{dP}{d\theta} = 2R$$

Substitute in eqn (1)

$$2R\theta\left[R^2(1-\cos 2\theta)\right] - R^2\left[\theta - \frac{\sin 2\theta}{2}\right]\cdot 2R = 0$$

$$2R^3\left[[1-\cos 2\theta]\theta - \theta + \left[\frac{\sin 2\theta}{2}\right]\right] = 0$$

$$\cancel{\theta} - \theta\cos 2\theta - \cancel{\theta} + \frac{\sin 2\theta}{2} = 0$$

$$\frac{\sin 2\theta}{\cos 2\theta} = 2\theta$$

$$\tan 2\theta = 2\theta$$

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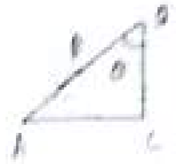
By hit and trial method

$$\boxed{\theta = 128^\circ 45'}$$



$d = OD - OC$

$= R - R\cos\theta$

$d = R[1-\cos\theta]$

$\cos\theta = \frac{OC}{R}$

$OC = R\cos\theta$

$d = \frac{D}{2}\left[1 - \cos 128^\circ 45'\right]$

$= \frac{1.625 D}{2} = 0.81D.$

$$\boxed{d = 0.81D}$$

$m = \frac{A}{P} = \frac{R^2\left[\theta - \frac{\sin 2\theta}{2}\right]}{2R\theta}$

$= \frac{R\left[2.247 - \frac{\sin 2(128^\circ 45')}{2}\right]}{2 \times 2.247}$

θ in rad $= 2.247.$

$= \frac{D}{2} \times 0.61$

$$\boxed{m = 0.3D}$$

Thus for max velocity depth = 0.81 times Diameter and hydraulic depth = 0.3 times Diameter

Condition for maximum discharge.

$Q = AC\sqrt{mi}$

$= AC\sqrt{\frac{A}{P} i}$

$= C\sqrt{\left[\frac{A^3}{P}\right] i}$

for max discharge. $\frac{d\left[\frac{A^3}{P}\right]}{d\theta} = 0.$

$\frac{P \cdot \frac{d(A^3)}{d\theta} - A^3 \cdot \frac{dP}{d\theta}}{P^2} = 0.$

$$P \cdot 3A^2 \frac{dA}{d\theta} - A^3 \cdot \frac{dP}{d\theta} = 0$$

$$\div A^2$$

$$3P \cdot \frac{dA}{d\theta} - A \frac{dP}{d\theta} = 0$$

$$3(2R\theta) \; R^2\left[1 - \cos 2\theta\right] - \left[R^2\left(\theta - \frac{\sin 2\theta}{2}\right)\right] \cdot 2R = 0$$

$$6R^3\theta\,(1 - \cos 2\theta) - 2R^3\left(\theta - \frac{\sin 2\theta}{2}\right) = 0$$

$$3\theta - 3\theta\cos 2\theta - \theta + \frac{\sin 2\theta}{2} = 0$$

$$2\theta - 3\theta\cos 2\theta + \frac{\sin 2\theta}{2} = 0$$

$$4\theta - 6\theta\cos 2\theta + \sin 2\theta = 0.$$

By hit and trial method

$$\boxed{\theta = 154^\circ}$$

$$d = OD - OC = R - R\cos\theta.$$

$$= \frac{D}{2}\left[1 - \cos 154^\circ\right]$$

$$\boxed{d = 0.95D}.$$

The rate of flow of water through a circular channel of diameter 0.6m is 150lps. Find the slope of the bed of the channel for max velocity. Take $c = 60$.

$Q = 150\times10^{-3}\ m^3/s$

$i = ?$

$C = 60$

$D = 0.6$

For max velocity

$$\theta = 128^\circ 45' = 2.24 \text{ radians}$$

$$d = 0.81D.$$

$$m = 0.3D.$$

$$P = 2R\theta = 2\times0.3\times2.24 = 1.3482\,m$$

$$A = R^2\left[\theta - \frac{\sin 2\theta}{2}\right] = 0.3^2\left[2.24 - \frac{\sin(2\times128^\circ45')}{2}\right]$$

$$= 0.242\,m^2.$$

$$m = 0.18\,m.$$

$$Q = AC\sqrt{mi}$$

$$150\times10^{-3} = 0.242\times60\sqrt{0.18\times i}$$

$$\boxed{i = 1694}$$

Determine the maximum discharge of water through a circular channel of diameter 1.5m when the bed slope of the channel is 1 in 1000. Take C = 60.

$D = 1.5m$

$R = \frac{1.5}{2} = 0.75m$

$i = 1/1000$

$C = 60$

For maximum discharge, $\theta = 154° \times \frac{\pi}{180} = 2.68$ radians

$P = 2R\theta = 2 \times 0.75 \times 2.68$

$= 4.0317\ m$

$A = R^2\left[\theta - \frac{\sin 2\theta}{2}\right]$

$= 0.75^2\left[2.68 - \frac{\sin(2\times 154°)}{2}\right]$

$= 1.7335\ m^2$

$m = \frac{A}{P} = \frac{1.7335}{4.0317} = 0.4299$

$Q = AC\sqrt{mi} = 1.7335 \times 60\sqrt{0.43 \times \frac{1}{1000}}$

$Q = 2.1565\ m^3/s$

A circular concrete lined channel of diameter 3m has a bed slope of 1 in 500. Work out the velocity and flow of rate for the conditions (i) max velocity (ii) max discharge.

Max Velocity.

$\theta = 128°45'$

$\theta = 128°45' \times \frac{\pi}{180} = 2.25$ rad.

$P = 2R\theta = 2 \times 1.5 \times 2.25 = 6.741m$

$A = R^2\left[\theta - \frac{\sin 2\theta}{2}\right] = 1.5^2\left[2.25 - \frac{\sin 2(128°45')}{2}\right]$

$= 6.1537\ m^2$

$m = \frac{6.15}{6.74} = 0.912$

$$V = c\sqrt{mi} = 50\sqrt{0.912 \times \frac{1}{500}} = 2.135 \, m/s$$

$$Q = A \times V = 6.1537 \times 2.135$$
$$= 13.138 \, m^3/s$$

For Maximum Discharge

$$\theta = 154^\circ = 154 \times \frac{\pi}{180} = 2.6878 \text{ rad}$$

$$A = R^2\left[\theta - \frac{\sin 2\theta}{2}\right] = 1.5^2\left[2.6878 - \frac{\sin(2 \times 154)}{2}\right]$$

$$= 6.934 \, m^2$$

$$P = 2R\theta = 2 \times 1.5 \times 2.6878$$
$$= 8.0634$$

$$m = \frac{A}{P} = 0.859$$

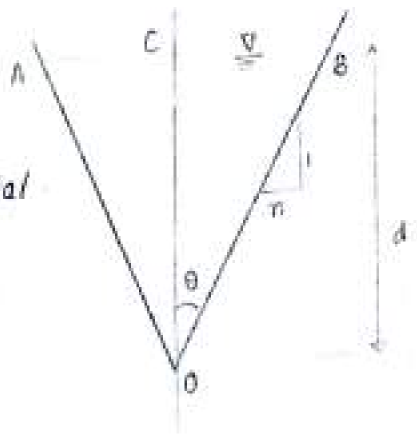
$$V = c\sqrt{mi} = 50 \times \sqrt{0.859 \times \frac{1}{500}}$$
$$= 2.0725 \, m/s$$

$$Q = Ac\sqrt{mi} = 6.934 \times 50 \times \sqrt{0.859 \times \frac{1}{500}}$$
$$= 14.377 \, m^3/s$$

Most economical Triangular Channel Section.

Consider a triangular channel section with depth of flow 'd' and side slopes 1 vertical to n horizontal

θ = angle made by the sides with the vertical



From Δ^{le} COB

$$\tan\theta = \frac{CB}{OC}$$

$$\tan\theta = \frac{CB}{d}$$

$$d\tan\theta = CB$$

$$\cos\theta = \frac{OC}{OB}$$

$$OB = \frac{1}{\cos\theta}$$

$$\boxed{OB = d\sec\theta}$$

Area of Δ^{n} section $= \frac{1}{2} \times AB \times OC$

$$= \frac{1}{2} \times 2d\tan\theta \times d$$

$$A = d^2 \tan\theta$$

$$d = \sqrt{\frac{A}{\tan\theta}}$$

$$P = 2d\sec\theta$$

differentiate P with respect to θ and equate it to zero.

$$\frac{dP}{d\theta} = 2\left[\frac{\sqrt{A}}{\sqrt{\tan\theta}}\right] \cdot \sec\theta$$

$$= 2\sqrt{A}\frac{d}{d\theta}\left[\frac{\sec\theta}{\sqrt{\tan\theta}}\right]$$

$$0 = 2\sqrt{A}\left[\frac{\sqrt{\tan\theta}\,(\sec\theta\tan\theta) - \frac{\sec\theta(\tan\theta)^{-1/2}}{2}\sec^2\theta}{(\sqrt{\tan\theta})^2}\right]$$

$$= \cancel{\frac{\sec\theta}{\sqrt{\tan\theta}}} - \cancel{\frac{\sec^3\theta}{2}}$$

$$0 = \sqrt{\tan\theta}\,\sec\theta - \frac{\sec^3\theta}{2(\sqrt{\tan\theta})\tan\theta}$$

$$0 = \sqrt{\tan\theta}\cdot\sec\theta - \frac{\sec^3\theta}{2\sqrt{\tan\theta}\,\tan\theta}$$

$$0 = \sec\theta\left[2\tan^2\theta - \frac{\sec^3\theta}{2\tan^{3/2}}\right]$$

$$\sec\theta \neq 0.$$

$$2\tan^2\theta - \sec^3\theta = 0$$

$$2\tan^2\theta = \sec^3\theta$$

$$\frac{2\sin^2\theta}{\cos^2\theta} = \frac{1}{\cos^3\theta}$$

$$\sin^2\theta = \frac{1}{2}$$

$$\sin\theta = \frac{1}{\sqrt{2}}$$

$$\theta = \sin^{-1}\left[\frac{1}{\sqrt{2}}\right]$$

$$\boxed{\theta = 45^\circ}$$

$$A = d^2\tan 45^\circ = d^2$$

$$P = 2d\sec 45^\circ = 2.82d$$

$$m = \frac{d^2}{2.82d} = 0.35d$$

$$\boxed{m = 0.35d}$$

Find the max discharge through a triangular channel whose bed slope is 1 in 3000. Take $C = 55$ and depth of flow is 3 m.

$Q = ?$

$i = 1/3000$

$C = 55$

$d = 3m$

Condition for max discharge.

$\theta = 45^\circ$,

$$\cos\theta = \frac{AC}{OC}$$

$$A = \frac{1}{2} \times AB \times OC$$

$$= \frac{1}{2} \times 2d\tan\theta \times d$$

$$A = d^2\tan\theta$$

$$\tan\theta = \frac{AC}{OC}$$

$$A = (3)^2\tan 45^\circ$$

$$AC = d\tan\theta$$

$$= 9m^2$$

$$AB = 2d\tan\theta$$

$$P = 2d\sec\theta = 2(3)\sec 45^\circ$$

$$= 8.48\ m.$$

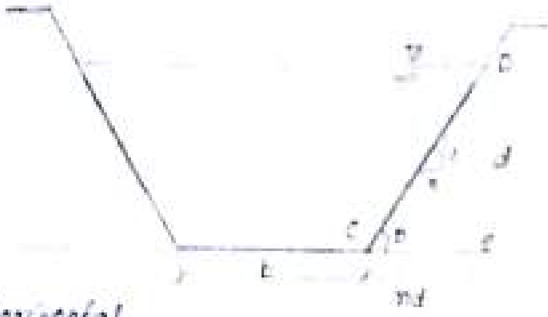
$$m = \frac{A}{P} = \frac{9}{8.48} = 1.06\,m$$

$$Q = AC\sqrt{mi}$$

$$= 9 \times 55 \sqrt{1.06 \times \frac{1}{3000}}$$

$$= \underline{9.31}\ m^3/s$$

Most economical Trapezoidal Channel.



Consider a trapezoidal section of channel as shown in fig

b = width of the channel

d = depth of flow

θ = angle made by the sides with horizontal

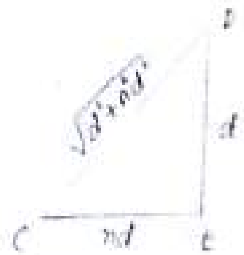
Side slope is given as 1 vertical to n horizontal.

Area of flow $A = (b \times d) + 2 \times \left[\frac{1}{2} \times nd \times d\right]$

$$A = bd + nd^2$$

$$A = d(b+nd) \quad \text{——— (1)}$$

$$b = \frac{A}{d} - nd \quad \text{——— (2)}$$



$$P = b + 2\sqrt{d^2(n^2+1)}$$

$$P = b + 2d\sqrt{n^2+1}$$

for economical section, $\frac{dP}{d(d)} = 0$

$$P = \frac{A}{d} - nd + 2d\sqrt{n^2+1}$$

$$\frac{dP}{d(d)} = -\frac{A}{d^2} - n + 2\sqrt{n^2+1}$$

$$\frac{A}{d^2} + n = 2\sqrt{n^2+1}$$

$$\frac{d(b+nd)}{d^2} + n = 2\sqrt{n^2+1}$$

$$\frac{b+nd}{d} + n = 2\sqrt{n^2+1}$$

$$b + nd + nd = 2d\sqrt{n^2+1}$$

$$\boxed{\frac{b+2nd}{2} = d\sqrt{n^2+1}} \quad \text{——— (1) condition}$$

half of top width = one of the sloping side

Hydraulic mean depth = $m = \frac{A}{P}$

$$= \frac{(b+nd)d}{b+2d\sqrt{n^2+1}}$$

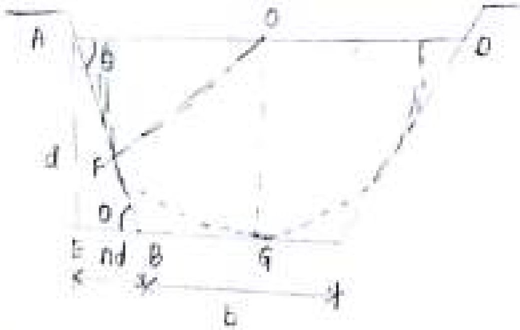
from condition (1)

$$= \frac{(b+nd)d}{b+b+2nd} = \frac{d(b+nd)}{2(b+nd)}$$

$$\boxed{m = d/2}$$

Hydraulic mean depth is equal to half the depth of flow.

Let the fig shows the trapezoidal channel of most economic section



θ = angle made by the sloping side with horizontal.

O = the centre of the top width.

Draw OF ⊥ to the sloping side AB.

From Δ^{le} AOF, $\sin\theta = \frac{OF}{OA}$ ——— ①

$OA \sin\theta = OF$

In Δ^{le} ACB $\sin\theta = \frac{AE}{AB} = \frac{d}{d\sqrt{n^2+1}}$

$$\sin\theta = \frac{1}{\sqrt{n^2+1}}$$

$$OA \times \frac{1}{\sqrt{n^2+1}} = OF$$

$$\frac{b+2nd}{2} \times \frac{1}{\sqrt{n^2+1}} = OF$$

$$\frac{d\sqrt{n^2+1}}{\sqrt{n^2+1}} = OF$$

OF = d, depth of flow.

If a semicircle is drawn with O as centre and radius equal to depth of flow, the 3 sides of section will be tangential to the semi circle

A trapezoidal channel has side slopes of 1H to 2 vertical and the slope of the bed is 1 in 1500. The area of the section is $40m^2$. Find the dimensions of the section if it is most economical. Determine the discharge of the most economical section if C = 50.

$$n = \frac{\text{Horizontal}}{\text{Vertical}}$$

$$n = \frac{1}{2}$$

$i = 1/1500$
$A = 40m^2$
$C = 50$

for economical section.

$$\frac{b+2nd}{2} = d\sqrt{n^2+1}$$

$$\frac{b+2\left(\frac{1}{2}\right)d}{2} = d\sqrt{\left[\frac{1}{4}\right]+1}$$

$$b+d = 2d(1.118)$$

$$\boxed{b = 1.236d}$$

$$A = bd + \left(\frac{1}{2} \times nd \times d\right)2$$

$$= bd + nd^2$$

$$= (1.236d)d + \frac{1}{2}d^2$$

$$40 = 1.736d^2$$

$$d^2 = \frac{40}{1.736}$$

$$d = 4.80 m$$

$$b = 1.236 \times 4.80$$

$$= 5.94 m$$

$$m = \frac{d}{2} = \frac{4.80}{2} = 2.40m$$

$$Q = AC\sqrt{mi} = 40 \times 50\sqrt{2.4 \times \frac{1}{1500}}$$

$$= \underline{80 m^3/s}$$

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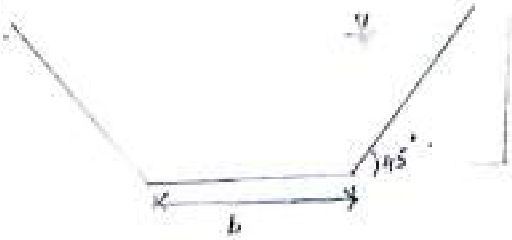
A trapezoidal channel to carry $160 m^3/min$ of water is designed to have a min cross section. Find the bottom width and depth if the bed slope is 1 in 1400 the side slopes at $45°$ and $C = 55$.

$Q = 160\ m^3/min = \frac{160}{60} = 2.67\ m^3/s$.



$i = \frac{1}{1400}$.

$\theta = 45°$.

$\tan\theta = \frac{1}{n}$

$n = 1$

$C = 55$

from economical section.

$$\frac{b + 2nd}{2} = d\sqrt{n^2+1}$$

$$b + 2nd = 2d\sqrt{n^2+1}$$

$$b + 2d = 2d\sqrt{2}$$

$$\boxed{b = 0.828d}$$

$$Q = AC\sqrt{mi}$$

$$2.67 = (b + nd)d \times 55\sqrt{\frac{d}{2} \times \frac{1}{1400}}$$

$$2.67 = (0.828d + d)\, d \times 55\sqrt{\frac{d}{2800}}$$

$$2.67 = \frac{1.828d^2 \times 55 \times d^{1/2}}{\sqrt{2800}}$$

$$d^{5/2} = 1.405$$

$$d = 1.14m.$$

$$b = 0.828 \times 1.14$$

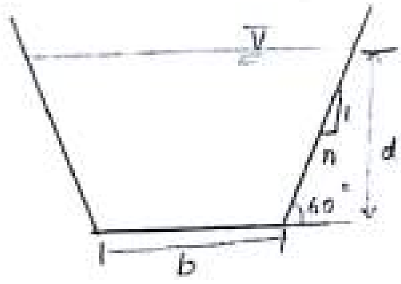
$$= \underline{0.948m}.$$

(11) A channel of most economical section has a slope of half hexagon with horizontal bottom to have maximum discharge of 24.4 m^3/s of water. The slope of the channel is 1 in 2800. Take C = 60. Determine the dimensions of the cross section.

$Q = 24.4 \ m^3/s$

$C = 60$

$i = 1/2800$



$\tan\theta = \tan 60° = \sqrt{3}$

$= \frac{1}{n}$

$n = \frac{1}{\sqrt{3}}$

$\tan\theta \ \frac{1}{n}$

Economical condition.

$$\frac{b+2nd}{2} = d\sqrt{n^2+1}$$

$$\frac{b+2\left(\frac{1}{\sqrt{3}}\right)d}{2} = d\sqrt{\left(\frac{1}{\sqrt{3}}\right)^2+1}$$

$$b+1.54d = 2d\sqrt{\frac{1}{3}+1}$$

$$b = (2.309-1.54)d$$

$$b = \cancel{0.769d} \cdot 1.154$$

$$Q = AC\sqrt{mi}$$

$$24.4 = b \times d + 2\left[\frac{1}{2} \times nd \times d\right] \times 60 \times \sqrt{\frac{d}{2} \times \frac{1}{2800}}$$

$$24.4 = \left[\left(\overset{1.154}{0.769} \times d^2\right) + \frac{1}{\sqrt{3}}d^2\right] \times 60 \times d^{1/2} \times \frac{1}{\sqrt{5600}}$$

$$24.4 = \frac{1.366d^2 \times 60 \times d^{1/2}}{\sqrt{5600}}$$

$$22.28 = d^{5/2}$$

11.57 $\boxed{d = 3.46 \ m}$ 2.14

$b = 2.$

$= 3.54 m$

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A trapezoidal channel with side slopes of 1 to 1 has to be designed to carry 10 m³/s at a velocity of 2 m/s so that the amount of concrete lining for the bed and sides is the min. Calculate the area of lining required for one metre length of canal

$n = \frac{1}{1} = 1$

$Q = 10\ m^3/s$

$V = 2\ m/s$

$b, d = ?$

$P = ?$

$A = \frac{10}{2} = 5\ m^2$

for economical section

$\frac{b + 2nd}{2} = d\sqrt{n^2+1}$

$\frac{b + 2(1)d}{2} = d\sqrt{1+1}$

$b = 1.828d^2$ $\boxed{b = 0.828\,d}$

$A = bd + nd^2$

$5 = 1.828d^2 + 1(d^2)$

$5 =$

$A = bd + nd^2$

$5 = (0.828d)d + 1(d^2)$

$5 = 1.828d^2$

$\boxed{d = 1.653\ m}$

$\boxed{b = 1.369\ m}$

Area of lining required per metre length of canal $=$ Wetted Perimeter $\times$ length of canal

$= P \times 1$

$P = b + 2d\sqrt{n^2+1}$

$= 1.369 + 2 \times 1.653\sqrt{1+1}$

$P = 6.04\ m$

∴ Area of lining $= 6.04 \times 1 = \underline{6.047\ m^2}$

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Best side slope for most economical trapezoidal section

Wetted area $A = db + nd^2$

$b = \frac{A}{d} - \frac{nd^2}{d}$.

Wetted Perimeter $P = bd + 2d\sqrt{n^2+1}$

$P = \frac{A}{d} - nd + 2d\sqrt{n^2+1}$

for P to be min

$\frac{dP}{dn} = 0$.

$\frac{dP}{dn} = \frac{d}{dn}\left[\frac{A}{d} - nd + 2d\sqrt{n^2+1}\right]$

$\Rightarrow -d + 2d \times \frac{1}{2}(n^2+1)^{\frac{1}{2}-1} \times 2n = 0$

$= -d + \frac{2nd}{\sqrt{n^2+1}} = 0$

$\frac{2nd}{\sqrt{n^2+1}} = d$

$2n = \sqrt{n^2+1}$

Squaring on both sides

$4n^2 = n^2+1$

$\boxed{3n^2 = 1}$

$n = \frac{1}{\sqrt{3}}$

$\tan\theta = \frac{1}{n} = \sqrt{3}$

$\therefore \boxed{\tan\theta = 60^\circ}$

for most economical trapezoidal section

$$\frac{b + 2nd}{2} = d\sqrt{n^2+1}$$

$$\frac{b + 2\left(\frac{1}{\sqrt{3}}\right)d}{2} = d\sqrt{\left(\frac{1}{\sqrt{3}}\right)^2 + 1}$$

$$\frac{\sqrt{3}\,b + 2d}{2\sqrt{3}} = \frac{2d}{\sqrt{3}}$$

$$\sqrt{3}\,b + 2d = 4d$$

$$\sqrt{3}\,b = 2d$$

$$b = \frac{2d}{\sqrt{3}}$$

$$P = b + 2d\sqrt{n^2+1}$$

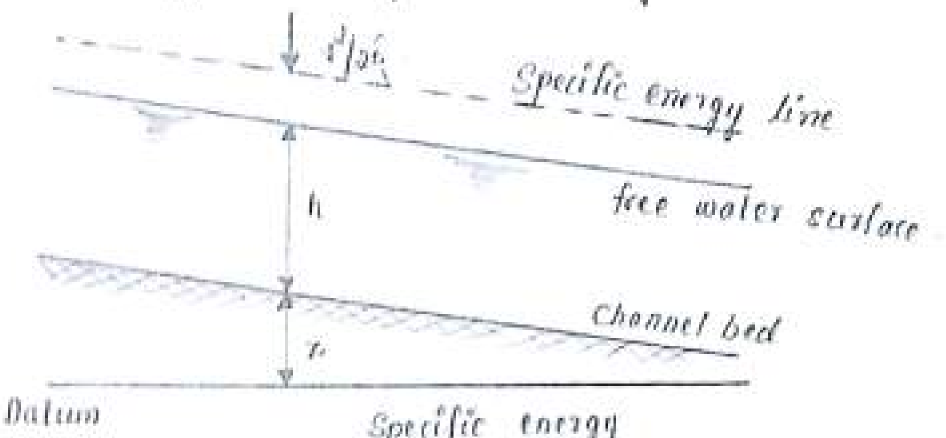
$$= \frac{2d}{\sqrt{3}} + 2d\sqrt{\left(\frac{1}{\sqrt{3}}\right)^2 + 1}$$

$$= \frac{2d}{\sqrt{3}} + 2d \cdot \frac{2}{\sqrt{3}}$$

$$= \frac{6d}{\sqrt{3}} = \frac{3 \times 2d}{\sqrt{3}}$$

$$\boxed{P = 3b}$$

Specific Energy and Specific Energy Curve



Specific energy

The total energy of a flowing liquid per unit weight is given by,

$$\text{Total energy} = z + h + \frac{V^2}{2g}$$

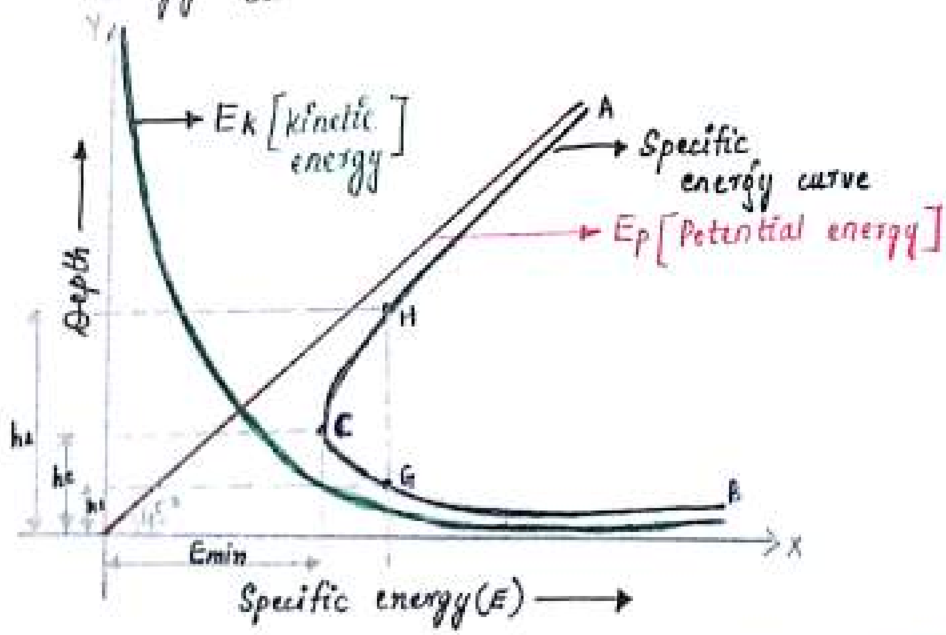
where z = height of the bottom of channel above datum.
h = Depth of liquid
V = mean velocity of flow.

If the channel bottom is taken as the datum, then the total energy per unit/weight of liquid will be,

$$E = h + \frac{V^2}{2g}$$

The energy E is known as Specific energy.

Specific Energy Curve



Specific energy of a flowing fluid is

$$E = h + \frac{V^2}{2g} = E_p + E_k$$

E_p = potential energy $= h$

E_k = kinetic energy $= \frac{V^2}{2g}$

Consider a rectangular channel in which a steady but non uniform flow is taking place.

Q = discharge through the channel

b = width of the channel

h = depth of flow

q = discharge per unit width

$$q = \frac{Q}{width} = \frac{Q}{b} = \text{constant}$$

velocity of flow, $V = \frac{Discharge}{Area} = \frac{Q}{b \times h} = \frac{q}{h}$

$$E = h + \frac{q^2}{2gh^2} = E_p + E_k.$$

The specific energy curve may also be obtained by first drawing a curve for potential energy, which will be a straight line passing through the origin making an angle of 45° with the X axis. Then drawing another curve for kinetic energy which will be parabolic.

By combining these 2 curves, we can obtain the specific energy curve.

Critical Depth (hc). It is defined as that depth of flow of water at which the specific energy is minimum. This is denoted by 'h_c'. In the fig, the curve ACB is a specific energy curve and point C corresponds to minimum specific energy. The depth of flow of water at C is known as Critical depth.

$$h_c = \left[\frac{q^2}{g}\right]^{1/3}.$$

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Critical Velocity (V_c) : The velocity of flow at the critical depth is known as critical velocity. It is denoted as V_c

$$V_c = \sqrt{g \times h_c}$$

Minimum specific energy in terms of Critical Depth

$$E_{min} = \frac{3h_c}{2}$$

Critical flow : It is defined as that flow at which the specific energy is minimum.

Froude number $F_e = 1.0$ for critical flow

Sub Critical flow : When the depth of flow in a channel is greater than the critical depth, the flow is said to be subcritical or tranquil or streaming flow. Froude's number is less than 1 in a sub critical flow.

$$F_e < 1.0$$

Super Critical flow : When the depth of flow in a channel is lesser than the critical depth, the flow is said to be supercritical or shooting flow or torrential flow. Froude's number is greater than 1 in a super critical flow.

$$F_e > 1.0$$

Condition for max discharge for a given value of Specific energy.

$$E = \frac{3h}{2}$$ and h is the critical depth

i.e. $$E = \frac{3h_c}{2}$$

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Q. Find the specific energy of flowing water through a rectangular channel of width 5m when the discharge is $10 m^3/s$ and depth of water is 3m.

Soln :

$$E = h + \frac{V^2}{2g}$$

$h = 3m$

$b = 5m$

$Q = 10 m^3/s$.

$$Q = A \times V$$

$$V = \frac{Q}{A} = \frac{10}{3 \times 5} = \frac{2}{3}$$

$$E = 3 + \frac{[2/3]^2}{2 \times 9.81} = \underline{3.0226\ m}$$

Q. Find the critical depth and critical velocity of the water flowing through a rectangular channel of width 5m, when the discharge is $15 m^3/s$.

Soln : $h_c = ?$

$V_c = ?$

$Q = 15 m^3/s$

$b = 5m$

$$h_c = \left[\frac{q^2}{g}\right]^{1/3}$$

$$V_c = \sqrt{g \times h_c}$$

$$q = \frac{Q}{b} = \frac{15}{5} = 3 m^2/s$$

$$h_c = \left[\frac{3^2}{9.81}\right]^{1/3}$$

$$\boxed{h_c = 0.972m}$$

$$V_c = \sqrt{9.81 \times 0.972}$$

$$\boxed{V_c = 3.088\ m/s}$$

Alternate Depths : In the specific energy curve, for any other value of specific energy, there are 2 depths, one greater than the critical depth and one smaller. These 2 depths for a given specific energy are called alternate depths ie h_1 and h_2.

Q The discharge of water through a rectangular channel of width 8m is 15 m^3/s. when the depth of flow of water is 1.2m Calculate a> Specific energy b> Critical depth c> Critical velocity d> Value of minimum specific energy

Soln: $Q = 15\,m^3/s$

$b = 8m$

$d = 1.2m$

$Q = A \times V$

$V = \frac{Q}{A}$

$= \frac{15}{(8 \times 1.2)}$

$= 1.5625\,m/s$

$q = \frac{Q}{b} = \frac{15}{8} = 1.875\,m^2/s$

i> $E = h + \frac{V^2}{2g}$

$= 1.2 + \frac{1.5625^2}{(2 \times 9.81)}$

$\boxed{E = 1.324m}$

ii> $h_c = \left[\frac{q^2}{g}\right]^{1/3}$

$= \left[\frac{1.875^2}{9.81}\right]^{1/3}$

$\boxed{h_c = 0.71m}$

iii> $V_c = \sqrt{g \times h_c}$

$= \sqrt{9.81 \times 0.71}$

$\boxed{V_c = 2.639\,m/s}$

iv> $E_{min} = \frac{3h_c}{2} = \frac{3 \times 0.71}{2}$

$\boxed{E_{min} = 1.065m}$

Q. The specific energy for a 5m wide rectangular channel is to be 4 Nm/N. If the rate of flow of water through the channel is $20 m^3/s$. Determine the alternate depths of flow.

Soln :

$b = 5m$

$E = 4m$

$Q = 20 m^3/s$

$V = \frac{Q}{A}$

$= \frac{20}{h \times b}$

$= \frac{20}{5 \times h}$

$V = \frac{4}{h}$

$E = h + \frac{V^2}{2g}$

$4 = h + \frac{V^2}{[2 \times 9.81]}$

$4 = h + \frac{16/h^2}{2 \times 9.81}$

$4 = h + \frac{0.815}{h^2}$

$-4h^2 + h^3 + 0.815 = 0$

$h^3 - 4h^2 + 0.815 = 0$

$\boxed{h_1 = 3.947 m}$

$\boxed{h_2 = 0.48 m}$

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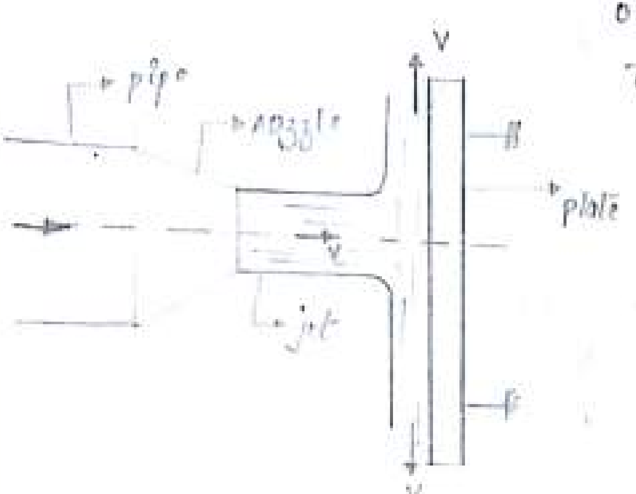
Impact of Jet on Vanes

The liquid which comes out from the outlet of a nozzle which is fitted to a pipe will flow under pressure. If some plate moving or stationary is placed in the path of jet, a force is exerted by the jet on the plate. This force is obtained from Newton's second law of motion or from impulse momentum equation.

Impact of jet means the force exerted by the jet on a plate either moving or stationary.

Force exerted by the jet on a Stationary Vertical Plate.



Consider a jet of water coming out from the nozzle, strikes a flat vertical plate.

Let v = velocity of the jet

d = diameter of the jet

a = area of cross section of jet $= \frac{\pi d^2}{4}$

The jet strikes the plate and moves along the plate. The jet gets deflected through 90° hence the component of the velocity of jet in the direction of jet is zero.

By impulse momentum equation or Newton's second law.

The force exerted by the jet on the plate in the direction of force $= \dfrac{\text{Initial momentum} - \text{final momentum}}{\text{time}}$

$= \dfrac{\text{mass} \times \text{Initial velocity} - \text{mass} \times \text{final velocity}}{\text{time}}$

$= \dfrac{m[v_1 - v_2]}{\text{time}}$

$\frac{mass}{sec}[v - 0]$

$\rho \times \frac{volume}{sec} \times velocity$

$= \rho \times Q \times v$

$= \rho \times a \times v \times v$

$\boxed{F_x = \rho \times a \times v^2}$

If the force exerted by the plate on the jet has to be calculated, then final velocity minus initial velocity has to be taken.

→ Find the force exerted by a jet of water of diameter 75 mm on a stationary flat plate, when the jet strikes the plate normally with velocity of 20 m/s

$F_x = ?$
$d = 75 \times 10^{-3}\,m$
$V = 20\,m/s$

$$F_x = \rho a v^2$$

$$= (1000) \times \left(\frac{\pi \times (75 \times 10^{-3})^2}{4}\right) \times 20^2$$

$$= 1766.8\,N$$

Force exerted by a jet on stationary Inclined Flat plate

Consider a jet of water coming out from the nozzle striking a inclined flat plate

Let v = velocity of the jet of water
d - diameter of the jet
a = cross sectional area of the jet
θ = angle between plate and the jet

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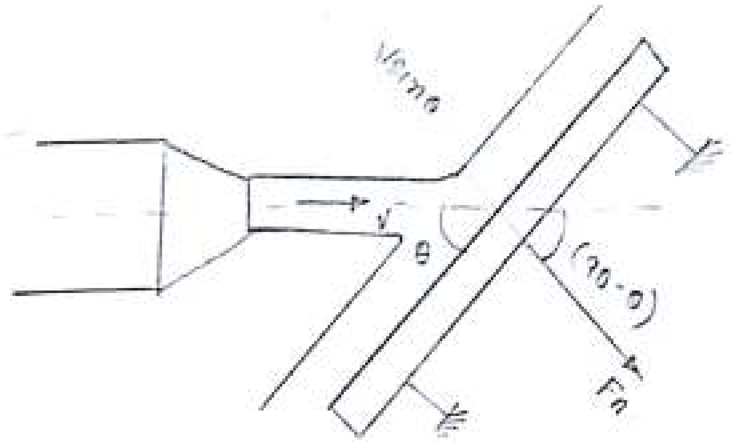


The force exerted by the jet on the plate in the direction normal to the plate has to be found. Let this force is represented by F_n.

Then F_n = mass of jet striking per sec $\left[\begin{array}{c}\text{initial velocity - final velocity} \\ \text{in the direction of } n\end{array}\right]$

$$= \rho a v [V \sin\theta - 0] = \rho a v^2 \sin\theta.$$

This force has to be resolved into X and Y components.

$$F_x = F_n \cos(90^\circ - \theta) = \rho a v^2 \sin\theta (\sin\theta)$$

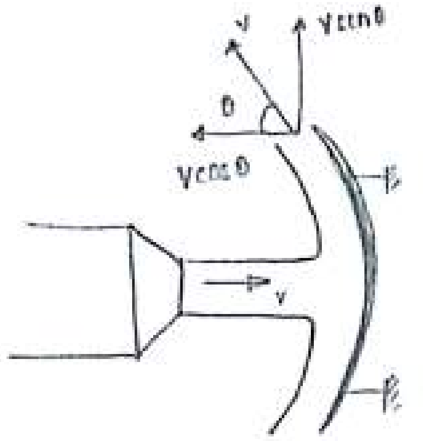
$$F_x = \rho a v^2 \sin^2\theta$$

$$F_y = F_n \sin(90 - \theta) = \rho a v^2 \sin\theta (\cos\theta)$$

$$F_y = \rho a v^2 \sin\theta \cos\theta$$

Force exerted by a jet on Stationary Curved Plate at the centre.

Consider a jet of water striking a fixed curved plate at the centre as shown.



Force exerted in the direction of jet.

$F_x = \rho a v [v - (-v\cos\theta)]$

$= \rho a v [v + v\cos\theta]$

$= \rho a v^2 (1 + \cos\theta]$

Force exerted in the y direction

$F_y = \rho a v [0 - v\sin\theta]$

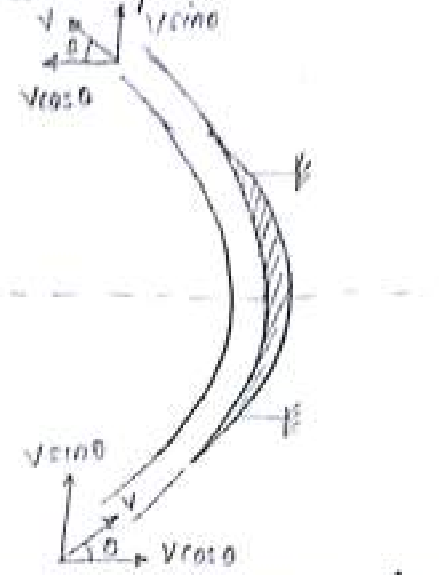
$= \rho a v (-v\sin\theta]$

$= -\rho a v^2 \sin\theta$

-ve sign indicates that the force is acting in the downward direction.

(3)

Force exerted by the jet of water striking on a curved fixed symmetrical plate at one end tangentially



Consider a jet striking a curved fixed plate at one end tangentially as shown in fig. Let the plate is symmetrical about x axis. Then the angle made by the tangents at both the ends of the plate will be same.

Let V = velocity of jet of water.

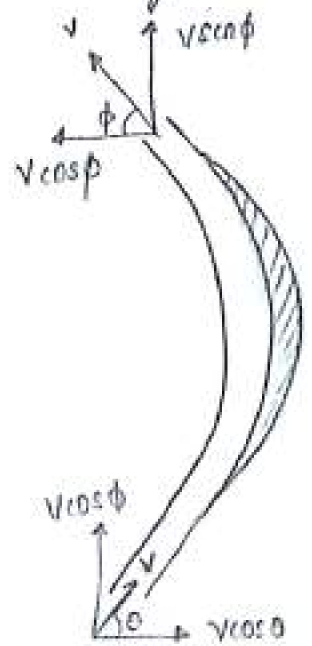
θ = angle made by jet with x axis at inlet tip of the curved plate.

The force exerted by the jet of water in x direction

$$F_x = \rho a V[V\cos\theta - (-V\cos\theta)]$$
$$= 2\rho a V^2 \cos\theta$$

$$F_y = \rho a V[V\sin\theta - V\sin\theta]$$
$$= \rho a V^2(0) = \underline{\underline{0}}$$

Force exerted on the curved plate when the jet strikes the plate at one end tangentially when the plate is unsymmetrical.



When the curved plate is unsymmetrical about x axis, the angles made by the tangents drawn at the inlet and outlet tips of the plate with x axis will be different.

Let θ = angle made by tangent at inlet tip with x axis

ϕ = angle made by tangent at outlet tip with x axis.

force exerted by the jet in the direction of jet $= F_x = \rho a v[v\cos\theta - (-v\cos\phi)]$

$$= \rho a v[v\cos\theta + v\cos\phi]$$

$$= \rho a v^2[\cos\theta + \cos\phi]$$

force exerted in Y direction

$$F_y = \rho a v[v\sin\theta - v\sin\phi$$

$$= \rho a v^2(\sin\theta - \sin\phi).$$

Water is flowing through a pipe at the end of which nozzle is fitted. The diameter of the nozzle is 100mm and the head of water at the centre of nozzle is 100m. Find the force exerted by the jet of water on a fixed vertical plate. The co-efficient of velocity is 0.95.

Soln $d = 100\times10^{-3}m$.

$h = 100m$.

$$V_{th} = C_v\sqrt{2gh}.$$
$$= 0.95\sqrt{2\times9.81\times100}$$
$$= \underline{42.07}\ m/s.$$

$$F_x = \rho a v^2$$
$$= 1000\times\frac{\pi(100\times10^{-3})^2}{4}\times42.07^2$$
$$= \underline{13907.2}N$$

A jet of water of diameter 75mm moving with a velocity of 25m/s strikes a fixed plate in such a way that the angle b/n the jet and plate is 60°. Find the force exerted by the jet on the plate (i) in the direction normal to the plate and (ii) in the direction of the jet.

Soln: $d = 75\times10^{-3}m$.

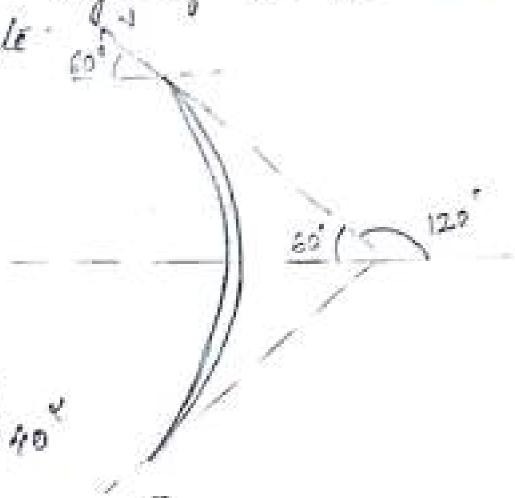
$V = 25 m/s$.

$\theta = 60°$.

$$F_n = \rho a v^2 \sin\theta$$
$$= 1000\times\frac{\pi(75\times10^{-3})^2}{4}\times25^2\times\sin60°$$
$$= \underline{2390.7N}$$

$$F_x = \rho a v^2 \sin^2\theta.$$
$$= 1000\times\frac{\pi(75\times10^{-3})^2}{4}\times25^2\times\sin^2 60$$
$$= \underline{2070.1\ N}$$

A jet of water of diameter 50mm moving with a velocity of 40 m/s, strikes a curved fixed symmetrical plate at the centre. Find the force exerted by the jet of water in the direction of jet, if the jet is deflected through an angle of 120° at the outlet of the curved plate.



Soln:

$d = 50 \times 10^{-3}\, m$

$V = 40\, m/s$

$\theta = 60°$

$$F_x = \rho a v^2 [1 + \cos\theta]$$

$$= 1000 \times \frac{\pi (50 \times 10^{-3})^2}{4} \times 40^2 \times [1 + \cos 60°]$$

$$= 4711.15 N$$

A jet of water of diameter 75mm moving with a velocity of 30 m/s strikes a curved fixed plate tangentially at one end at an angle of 30° to the horizontal. The jet leaves the plate at an angle of 20° to the horizontal. Find the force exerted by the jet on the plate in the horizontal and vertical direction.

Soln: $d = 75 \times 10^{-3}\, m$

$v = 30\, m/s$

$\theta = 30°$

$\phi = 20°$

$$F_x = \rho a v^2 [\cos\theta + \cos\phi]$$

$$= 1000 \times \frac{\pi (75 \times 10^{-3})^2}{4} \times 30^2 [\cos 30° + \cos 20°]$$

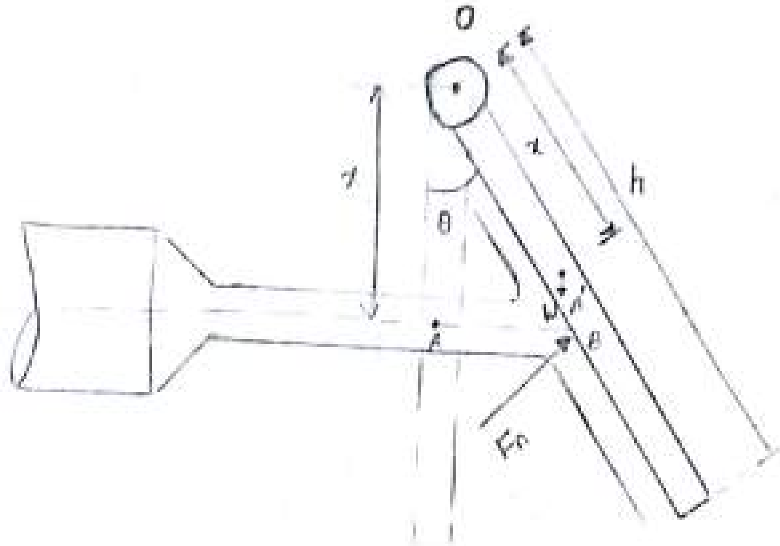
$$= 7178.2 N.$$

$$F_y = \rho a v^2 [\sin\theta - \sin\phi]$$

$$= 1000 \times \frac{\pi (75 \times 10^{-3})^2}{4} \times 30^2 [\sin 30 - \sin 20°]$$

$$= 628.13 N$$

Force exerted by a jet on a hinged plate.



Consider a jet of water striking a vertical plate at the centre which is hinged at O. Due to the force exerted by the jet on the plate, the plate will swing through some angle about the hinge.

Let x = distance of the centre of jet from hinge 'O'

θ = angle of swing.

W = weight of the plate acting at C.G of the plate

The dotted line shows the position of the plate before it strikes the plate. The point A will be A' after the jet strikes the plate. $\therefore$ OA = OA' = x.

Let weight of the plate which acts at CG is at A'.

The plate is in equilibrium if the moment of all the forces about the hinge is zero.

The forces acting on the plate are

(i) Force due to jet normal to the plate.

$$F_n = \rho a v^2 \sin\theta'$$

θ' = angle b/n jet and plate = $(90° - \theta)$

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$\Rightarrow$ weight of the plate W

Moment of force F_n about hinge $= F_n \times OB = \rho a v^2 \cos(90-\theta) \times OB$

$= \rho a v^2 \cos\theta \times \frac{OA}{\cos\theta} = \rho a v^2 x$

Moment of weight W about hinge $= W \times OA \sin\theta = W \times x \times \sin\theta$

$= Wx \sin\theta$

for equilibrium

$$\rho a v^2 x = Wx \sin\theta$$

$$\boxed{\sin\theta = \frac{\rho a v^2}{W}}$$

→ A jet of water of 2.5 cm diameter, moving with a velocity of 10 m/s, strikes a hinged square plate of weight 98.1 N at the centre of the plate. The plate is of uniform thickness. Find the angle through which the plate will swing.

Given

$d = 2.5 \times 10^{-2}$ m

$v = 10$ m/s

$W = 98.1$ N

$$\sin\theta = \frac{\rho a v^2}{W}$$

$$\sin\theta = \frac{1000 \times \frac{\pi (2.5 \times 10^{-2})^2}{4} \times 10^2}{98.1}$$

$\sin\theta = 0.499$

$\theta = \sin^{-1}(0.499)$

$$\boxed{\theta = 29^\circ}$$

→ A jet of water of 30mm dia strikes a hinged square plate at its centre with a velocity of 20m/s. The plate is deflected through an angle of 20° find the weight of the plate.

If the plate is not allowed to swing, what will be the force required at the lower edge of the plate.

Soln:

$d = 30\times10^{-3}$ m

$v = 20$ m/s

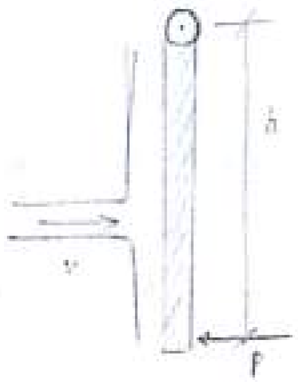
$\theta = 20^\circ$

$W = ?$

$$\sin\theta = \frac{\rho a v^2}{W}$$

$$\sin 20^\circ = \frac{\left(1000\times \frac{\pi(30\times10^{-3})^2}{4}\times 20^2\right)}{W}$$

$$\boxed{W = 826.6\,N}$$



A force P is applied at the lower edge as shown in the fig for the plate not to swing.

Let F = force exerted by jet of water.

h = height of plate

= Distance of P from the hinge.

The jet strikes the centre of plate and the distance of the centre of the jet from hinge = h/2.

Taking moments about the hinge

$$P\times h = F\times\frac{h}{2}$$

$$P = \frac{\rho a v^2}{2}$$

$$= \frac{1000\times \frac{\pi(30\times10^{-3})^2}{4}\times 20^2}{2}$$

$$\boxed{P = 141.36\,N}$$

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→ A square plate of uniform thickness and length of side 300mm hangs vertically from hinge at its top edge. When a horizontal water jet strikes the plate at its centre, the plate is deflected and comes to rest at an angle of 30° to the vertical. The jet is 25mm in dia and has a velocity of 6m/s. Determine the weight of the plate.

Soln $d = 25\times10^{-3}$ m

$u = 6$ m/s

$\theta = 30°$

$L = 300\times10^{-3}$ m

$W = ?$

$$W = \frac{\rho a v^2}{\sin\theta}$$

$$= \frac{\left(1000\times\pi\times\frac{(25\times10^{-3})^2}{4}\times 6^2\right)}{\sin 30°}$$

$$= \underline{\underline{35.33\,N}}$$

→ A jet of water of diameter 25mm strikes a 20cm × 20cm square plate of uniform thickness with a velocity of 10m/s at the centre of the plate which is suspended vertically by a hinge on its top horizontal edge. The weight of the plate is 98.1N. The jet strikes normal to the plate. What force must be applied at the lower edge of the plate so that plate is kept vertical? If the plate is allowed to deflect freely, what will be the inclination of the plate with vertical due to the force exerted by jet of water?

Sol^n : $d = 25\times10^{-3}$ m

$A = 20\times20$
$= 400\ cm^2$

$W = 98.1$ N

$P = ?$

$v = 10$ m/s

(i) Let the force applied at the lower edge to keep the plate in vertical position is P.

Force exerted by the jet of water at the centre $F = \rho a v^2$

$$= 1000 \times \frac{\pi (25\times10^{-3})^2}{4} \times 10^2$$

$$= \underline{49N}$$

The force is acting at a distance of $\frac{20}{2} = 10$ cm from hinge.

The force 'P' is acting at a distance of 20cm from the hinge

for equilibrium moments of the force about the hinge is zero

$F\times10 = P\times20$

$\frac{49\times10}{20} = P$

$\boxed{P = 24.5\ N}$

(ii) When the plate is allowed to deflect freely about hinge

Let the inclination with the vertical $= \theta$.

The angle between the plate and jet will be $= (90-\theta)$.

Force exerted by water normal to the plate

$$F_n = \rho a v^2 \sin(90-\theta) = \rho a v^2 \cos\theta$$

$$OB = \frac{OA}{\cos\theta} \Rightarrow \frac{10}{\cos\theta}$$

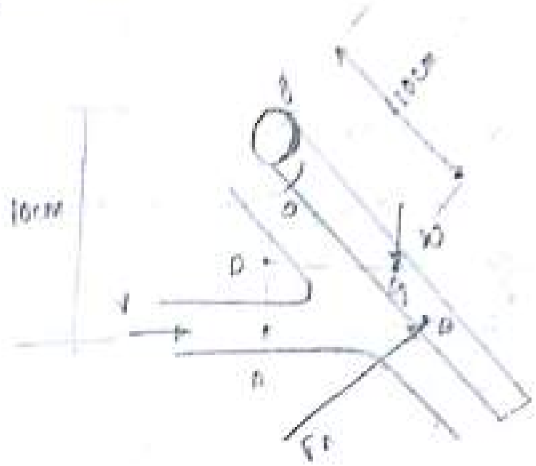
$$OG = OG \sin\theta = 10 \times \sin\theta$$

Taking moments about hinge

$$F_n \times OB = W \times GD$$

$$\rho a v^2 \cos\theta \times \frac{10}{\cos\theta} = W \times 10 \times \sin\theta$$

$$\theta = 20^{\circ}$$



A rectangular plate weighing 58.86 N is suspended vertically by a hinge on the top of horizontal edge. The C.G of the plate is 10 cm from the hinge. A horizontal jet of water 2 cm dia whose axis is 15 cm below the hinge impinges normally on the plate with a velocity of 5 m/s. Find the horizontal force applied at the centre of the gravity to maintain the plate in its vertical position. Find the corresponding velocity of the jet, if the plate is deflected through 30° and the same force continues to act at the C.G of the plate.

Given:

$W = 58.86\,N$

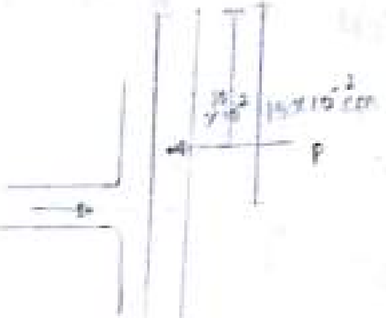
$d = 2\times10^{-2}\,m$

$v = 5\,m/s$

$$F_x = \rho a v^2$$

$$= \left(1000 \times \frac{(2\times10^{-2})^2\,\pi}{4} \times 5^2\right)$$

$$= \underline{7.85\,N}$$



Taking moment about the hinge

$$F_x \times 15\times10^{-2} = P\times10\times10^{-2}$$

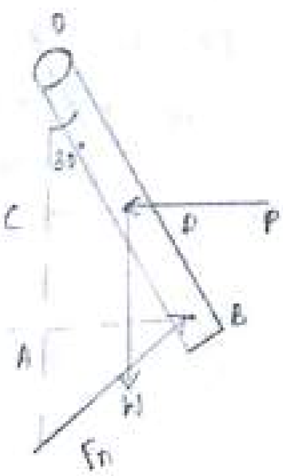
$$P = \underline{11.775\,N}$$

Three forces acting on the system are

1) Normal force F_n

2) weight of the plate

3) force P



$\cos30° = \frac{OC}{OD}$

$OC = 10\times10^{-2}\times\cos30°$

$\sin30° = \frac{CD}{10\times10^{-2}}$

$CD = 10\times10^{-2}\times\sin30°$

$\cos30° = \frac{15\times10^{-2}}{OB}$

$OB = \frac{15\times10^{-2}}{\cos30°}$

Taking moments about the hinge

$$-(P \times OC) + (W \times CD) - (F_n \times OB) = 0$$

$$-(11.775 \times 10 \times 10^{-2} \times \cos 30^\circ) + (58.86 \times 10 \times 10^{-2} \times \sin 30^\circ - \left(1000 \times \frac{\pi (20 \times 10^{-3})^2}{4} \times V^2 \cos^2 30^\circ \times \frac{15 \times 10^{-2}}{\cos 30^\circ}\right) -$$

$$V = \underline{9.175 \text{ m/s}}$$

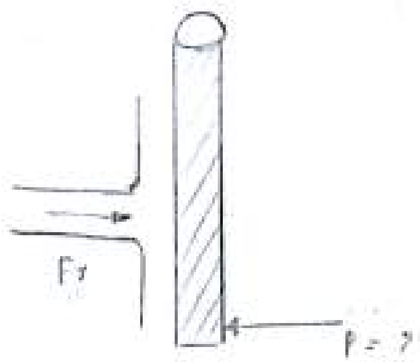
→ A jet of water of 30mm strikes 25 x 25 cm² square plate with a velocity 12 m/s at the centre of the hinged plate. The weight of the plate is 100 N. The jet strikes normal to the plate. What force must be applied at the lower edge of the plate so that plate is kept vertical? If the plate is allowed to deflect freely. what will be the inclination of the plate with vertical due to the force exerted by jet of water.

Soln : $d = 30 \times 10^{-3}$ m

$\ell = 25 \times 10^{-2}$ m.

$v = 12$ m/s

$W = 100$ N



$$F_x = \rho a v^2$$

$$= \left(1000 \times \pi \frac{(30 \times 10^{-3})^2}{4} \times 12^2\right)$$

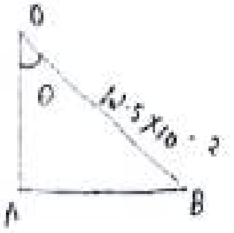
$$= 101.78 \text{ N}.$$

Taking moments about the hinge

$$F_x \times 12.5 \times 10^{-2} = P \times 25 \times 10^{-2}$$

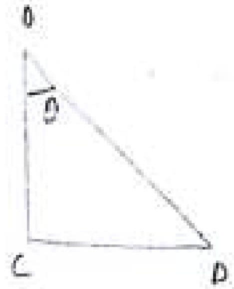
$$\frac{101.78 \times 12.5 \times 10^{-2}}{25 \times 10^{-2}} = P.$$

$$\underline{P = 50.89 \text{ N}}$$



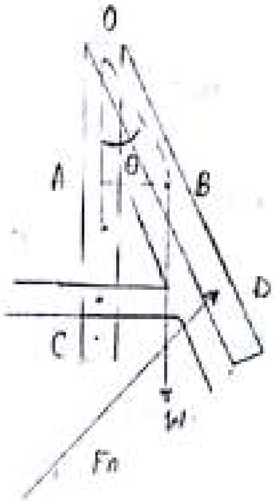
$\sin\theta = \frac{AB}{OB}$

$AB = 12.5\times10^{-2}\times\sin\theta$



$\cos\theta = \frac{OC}{OD}$

$OD = \frac{OC}{\cos\theta}$



Taking moment about the hinge 'O'

$F_n \times OD = W \times AB$

$\rho a v^2 \cos\theta \times \frac{OC}{\cos\theta} = 100\times 12.5\times10^{-2}\times\sin\theta$

$(1000)\left(\frac{\pi\times(30\times10^{-3})^2}{4}\right)\times 12^2 \times 12.5\times10^{-2} = 150\times12.5\times10^{-2}\times\sin\theta$

$1.01 = \sin\theta$

$12.723 =$

$\sin\theta = 0.67$

$\underline{\theta = 42^\circ 43'}$

→ A rectangular plate, weighing 60N is suspended vertically by a hinge on the top horizontal edge. The centre of gravity of the plate is 100mm from the hinge. A horizontal jet of water 20mm diameter, whose axis is 150mm below the hinge impinges normally on the plate with a velocity of 5m/s. Determine

(1) the horizontal force applied at the C.G to maintain the plate in its vertical position.

(2) The corresponding velocity of the jet, if the plate is deflected through 30° and the same force continues to act at C.G of the plate.

$W = 60N$

$x = 0.1m$

$d = 0.02m$

$a = \frac{\pi (0.02)^2}{4} = 0.000314 m^2$

$v = 15 m/s$

(i)

$F_x = \rho a v^2$

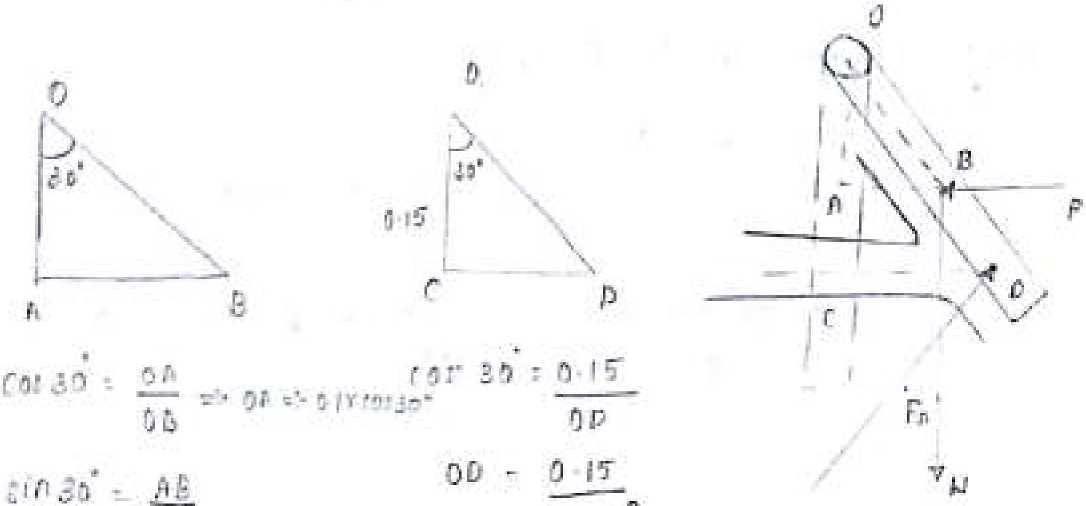
$= (1000 \times 0.000314 \times 15^2) = 7.85N$

Taking moments about the hinge

$F \times 0.15 = P \times 0.1$

$\underline{P = 11.77 N}$

(ii)



$\cos 30° = \frac{OA}{OB} \Rightarrow OA = 0.1 \times \cos 30°$

$\sin 30° = \frac{AB}{OB}$

$0.1 \sin 30° = AB$

$AB = 0.05m$

$\cos 30° = \frac{0.15}{OD}$

$OD = \frac{0.15}{\cos 30°}$

$OD = 0.1732m$

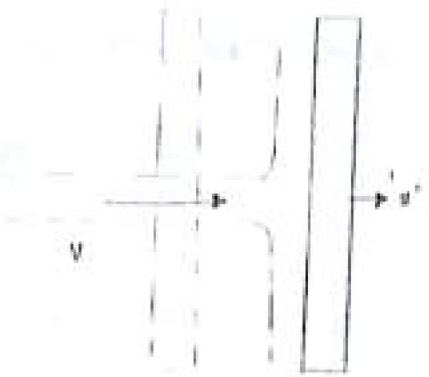
Taking moments about the hinge.

$-F_n \times 0.1732 + (P \times 0.1 \times \cos 30°) + (W \times 0.05) = 0.$

$-(\rho a v^2 \times 0.1732) + (11.77 \times 0.1 \times \cos 30°) + (60 \times 0.05) = 0.$

$v^2 = 73.9$ $v = 8.60 m/s$

Force exerted by the jet of water on a moving flat vertical plate in the direction of jet.



Consider a jet striking a flat vertical plate moving with a uniform velocity away from the jet.

Let v = velocity of the jet

u = velocity of the plate

a = cross sectional area of the jet.

In this case, the jet does not strike the plate with a velocity v, but it strikes with a relative velocity i.e $(v - u)$.

$\therefore$ Relative velocity of the jet with respect to plate

$= (v - u)$

Force exerted by the jet on the moving plate in the direction of the jet

$$F_x = \frac{\text{Initial momentum} - \text{Final momentum}}{\text{time}}$$

$$= \frac{\text{mass}}{\text{time}}\left[\text{initial velocity} - \text{final velocity}\right]$$

$$= \rho \times a (v - u) \left[(v-u) - 0\right]$$

$$F_x = \rho a (v-u)^2$$

Work done per sec by the jet on the plate

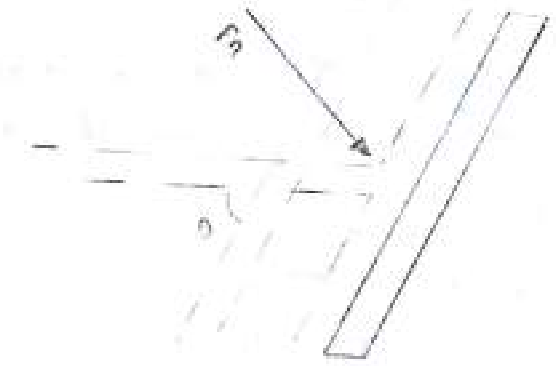
$= F_x \times u$

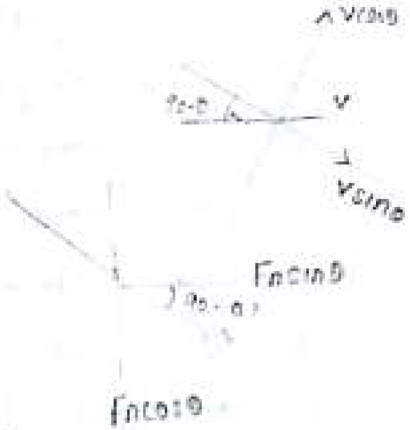
$= \rho a (v-u)^2 \times u$

Unit for work done per sec is $N\,m/s$ or 'Watt'.

Force on the inclined plate moving in the direction of the jet.

Let a jet of water strikes an inclined plate, which is moving with a uniform velocity in the direction of the jet.





Let v = absolute velocity of jet of water.

u = velocity of the plate in the direction of jet.

a = c/s area of jet.

θ = angle b/n jet and plate.

relative velocity of jet of water = $(v-u)$.

The velocity with which jet strikes = $(v-u)$

Force exerted by the jet on the plate in the direction normal to the plate.

$$F_n = \rho a (v-u)\left[(v-u)\sin\theta - 0\right]$$

$$= \rho a (v-u)^2 \sin\theta .$$

If this is resolved into x and y components

$F_x = F_n \sin\theta = \rho a (v-u)^2 \sin^2\theta$.

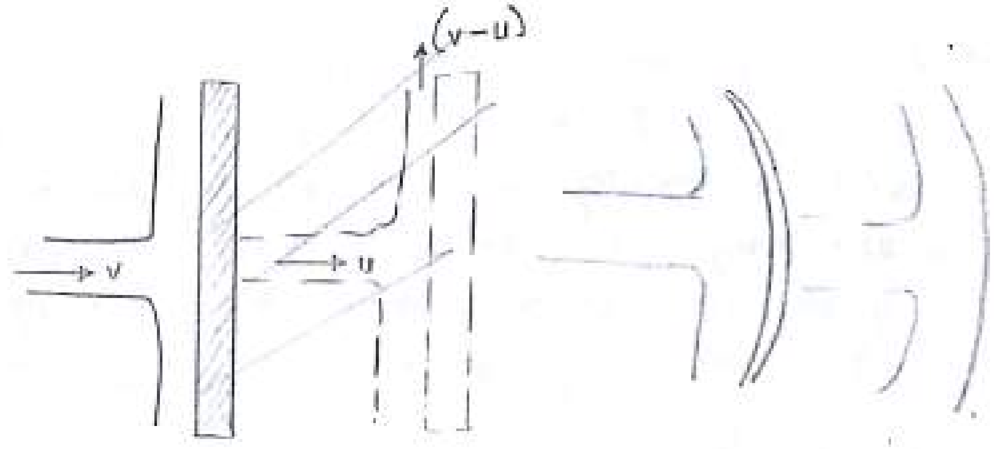
$F_y = F_n \cos\theta = \rho a (v-u)^2 \sin\theta \cos\theta$.

$\therefore$ Work done per second by the jet on the plate $= F_x \times u$

$= \rho a (v-u)^2 \sin^2\theta \times u$.

$= \rho a u (v-u)^2 \sin^2\theta$ N m/s.

Force exerted on the curved plate when the plate is moving in the direction of jet.



Consider a jet of water strikes a curved plate at the centre of the plate which is moving with a uniform velocity in the direction of the jet.

Let v = absolute velocity of jet

a = area of jet.

u = velocity of the plate.

relative velocity $= (v-u)$

If plate is smooth and energy loss is zero, the jet will leave the vane with a velocity $= (v-u)$.

This velocity is resolved into two components along x and y direction.

$V_x = -(v-u)\cos\theta$

$V_y = +(v-u)\sin\theta$

$$F_x = \rho a(v-u)\left[(v-u) + (v-u)\cos\theta\right]$$
$$= \rho a(v-u)^2\left[1+\cos\theta\right]$$

$$F_y = \rho a(v-u)\left[0 - (v-u)\sin\theta\right.$$
$$= -\rho a(v-u)^2 \sin\theta$$

Work done by the jet on the plate per sec $= F_x \times u$

$$= \rho a(v-u)^2\left[1+\cos\theta\right] \times u.$$
$$= \left[\rho a(v-u)^2 u\,(1+\cos\theta)\right].\ N\,m/s$$

→ A jet of water of diameter 150mm strikes a flat plate normally with a velocity of 12 m/s. The plate is moving with a velocity of 6m/s in the direction of the jet and away from the jet. Find (i) the force exerted by the jet on the plate (ii) work done by the jet on the plate per sec (iii) power of the jet (iv) efficiency of jet.

$d = 150 \times 10^{-3}\,m.$
$v = 12\ m/s$
$u = 6\ m/s.$

i) $F_x = \rho a(v-u)^2 = 1000 \times \frac{\pi (150\times10^{-3})^2}{4} \times [12-6]^2$

$$= 636.17\,N$$

ii) Work done by the jet on the plate per sec $= F_x \times u$

$$= 636.17 \times 6$$
$$= 3817.03\ N\,m/s.$$

iii) Power of the jet $= \frac{3817.03}{1000} = 3.817\ kW.$

iv) Efficiency $= \frac{O/P \text{ of jet per sec}}{I/P \text{ of jet per sec.}}$

O/P of jet / sec = work done by jet per sec = 3817.03 Nm/s.

I/P per sec = $\frac{\text{kinetic energy of the jet}}{\text{sec}}$ = $\frac{1}{2}(\rho a v) \times v^2$

$= \frac{1}{2}\left(\frac{mass}{sec}\right)v^2$

$= \frac{1}{2}\left(1000 \times \pi \frac{(150\times10^{-3})^2}{4} \times 12\right) \times 12^2$

=

η of jet = ———— = 25%.

→ A nozzle of 50mm diameter delivers a stream of water at 20m/s perpendicular to a plate that moves away from the jet at 5 m/s. Find (i) the force on the plate. (ii) the work done (iii) the efficiency.

Soln: $d = 50\times10^{-3}$ m.

$v = 20$ m/s.

$u = 5$ m/s.

i) force on the plate $= \rho a (v-u)^2$

$= \left(1000 \times \pi \frac{(50\times10^{-3})^2}{4} \times (20-5)^2\right.$

$= 441.78$ N.

ii) The work done by the jet $= F_x \times u$

$= 441.78 \times 5$

$= 2208.9$ Nm/s.

iii) Efficiency $\eta = \frac{\text{O/p of the jet / sec}}{\text{I/p of the jet / sec}}$.

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O/p of the jet /sec = Work done /s = 2208.7 Nm/s

I/p of the jet /sec = $\frac{1}{2}\left[\frac{mass}{sec}\right] \times v^2$

$= \frac{1}{2}[\rho a v] v^2$

$\eta = \frac{2208.7}{6540}$

33.77% $\quad$ 32.13%

→ A 7.5 cm diameter jet having a velocity of 30 m/s strikes a flat plate, the normal of which is inclined at 45° to the axis of the jet. Find the normal pressure on the plate (i) when the plate is stationary (ii) when the plate is moving with a velocity of 15 m/s and away from the jet. Also determine the power and efficiency of the jet when the plate is moving.

Soln:

$d = 7.5 \times 10^{-2}$ m.

$V = 30$ m/s

$\theta = 45°$

i) When the plate is stationary.

$$F_n = \rho a v^2 \sin\theta$$

$$= 1000 \times \frac{\pi (7.5\times10^{-2})^2}{4} \times \sin 45° \times 30^2$$

$$= 2810.96 \text{ N}$$

ii) When the plate is moving with a velocity of 15 m/s.

relative velocity $= (v - u) = (30 - 15) = 15$ m/s

$$F_n = \rho a (v-u)^2 \sin\theta$$

$$= 1000 \times \frac{\pi (7.5\times10^{-2})^2}{4} \times (15)^2 \times \sin 45°$$

$$= 702.74 \text{ N}$$

Work done by the jet per sec $= F_x \times u$

$= F_n \times \sin\theta \times u$

$= \rho a (v-u)^2 \sin^2\theta \times 15$

$= 7453.5$ Nm/s.

Power $= \frac{7453.5}{1000} = 7.45$ kW

$$\eta = \frac{O/P}{I/P} = \frac{7453.5}{\frac{1}{2}(\rho a v)v^2} = \frac{7453.5}{\frac{1}{2}\left[1000 \times \frac{\pi (0.075)^2}{4} \times 30^3\right]}$$

$= \underline{12.5\ \%}$

→ A jet of water of diameter 100mm strikes a curved plate at its centre with a velocity of 15 m/s. The curved plate is moving with a velocity of 7 m/s in the direction of the jet. The jet is deflected through an angle of 150°. Assuming the plate as smooth find (i) force exerted on the plate in the direction of the jet.
(ii) power of the jet (iii) efficiency.

Soln: $d = 100 \times 10^{-3}$ m

$v = 15$ m/s

$u = 7$ m/s.

$\theta = 180 - 150 = 30°$.

$F_x = ?$

$P = ?$

$\eta = ?$

$F_x = \rho a (v-u)^2 (1+\cos\theta)$

$= 1000 \times \frac{\pi (100 \times 10^{-3})^2}{4} \times (15-7)^2 (1+\cos 30°)$

$= \underline{938\ N}$

Work done by the jet / sec $= F_x \times u$

$= 938 \times 7$

$= 6565.76$ Nm/s.

Power $= 6.56$ kW

$$\eta = \frac{O/P}{I/P} \times 100$$

$$I/P = \frac{1}{2}(\rho a v) v^2$$

$$\eta = \frac{6565.76}{\frac{1}{2}\left[1000 \times \pi \frac{(100 \times 10^{-3})^2}{4} \times 15\right] 15^2}$$

$$= \underline{\underline{49.53\%}}$$

→ A jet of water from a nozzle is deflected through 60° from its original direction by a curved plate which it enters tangentially without shock with a velocity of 30 m/s and leaves with a mean velocity of 25 m/s. If the discharge from the nozzle is 0.8 kg/s. Calculate the magnitude and direction of the resultant force on the vane. if the vane is stationary.

Soln.

$V_1 = 30\ m/s$

$V_2 = 25\ m/s$

$\frac{mass}{sec} = 0.8\ kg/s$

$$F_x = 0.8[30 - (25 \cos 60)]$$
$$= \underline{14.0N}$$

$$F_y = 0.8[0 - 25 \sin 60] = -17.32N$$

$$R = \sqrt{F_x^2 + F_y^2} = \sqrt{14^2 + 17.32^2}$$

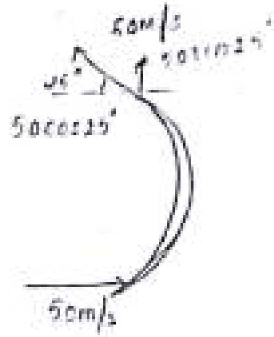
$$= 22.27N$$

$$\tan\theta = \frac{17.32}{14} = 1.237$$

$$\theta = \tan^{-1}(1.237)$$
$$= \underline{\underline{51^\circ 2.86'}}$$

(a) A stationary vane having an inlet angle of zero degree and an outlet angle of 25° as shown in fig, receives water at a velocity of 50m/s. Determine the components of force acting on it in the direction of the jet velocity and normal to it. Also find the resultant force in magnitude and direction per unit weight of flow.

(b) If the vane stated above is moving with a velocity of 20m/s in the direction of the jet. Calculate the force components in the direction of the vane velocity and across it, also the resultant force in magnitude and direction. Calculate the work done and power developed per unit weight of flow.



Soln (a)

$v = 50\ m/s$

$\theta = 25°$

$$F_x = \frac{mass}{sec}\left[50 - (-50\cos 25)\right]$$

$$F_x = \frac{mass}{sec}(95.315)$$

$$\frac{F_x}{\text{unit weight of flow}} = \frac{\frac{mass}{sec}(95.315)}{\frac{mass}{sec} \times g}$$

$W = \frac{m \times g}{s}$

$$= \underline{9.716\ N}$$

$$\frac{F_y}{\text{unit weight of flow}} = \frac{\frac{mass}{sec}\left[0 - 50\sin 25°\right]}{\frac{mass}{sec} \times g}$$

$$= \frac{-50\sin 25°}{9.81}$$

$$= -2.154$$

$$F_R = \sqrt{9.716^2 + 2.154^2}$$

$$= 9.952$$

$$\tan\theta = \frac{F_y}{F_x} = 12°$$

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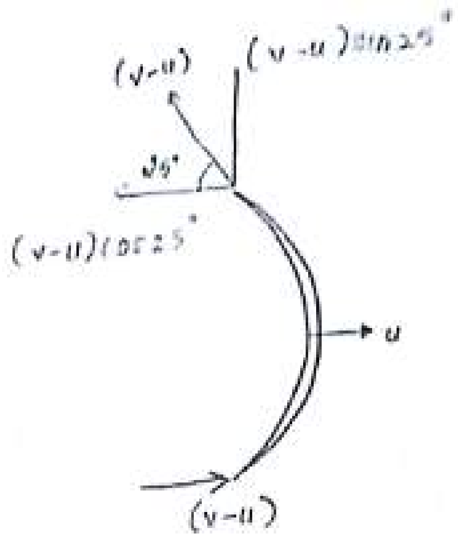
(b) $V = 20\, m/s$

$V = 50\, m/s$



$$F_x = \frac{mass}{sec}\left[(v-u) + (v-u)\cos 25^\circ\right]$$

$$= \frac{\frac{mass}{sec}\left[(30) + (30\cos 25^\circ)\right]}{\frac{mass}{sec} \times g}$$

$$= \frac{57.189}{9.81} = 5.83\, N.$$

$$F_y = \frac{mass}{sec}\left[0 - (v-u)\sin 25^\circ\right]$$

$$= \frac{mass}{sec}\left[-30\sin 25^\circ\right]$$

$$= \frac{\frac{mass}{sec}(-30\sin 25^\circ)}{\frac{mass}{sec} \times g} = -1.292\, N$$

$$R = \sqrt{5.83^2 + (1.292)^2}$$

$$= 5.917\, N$$

$$\theta = \tan^{-1}\left(\frac{1.292}{5.82}\right)$$

$$= 12.30^\circ.$$

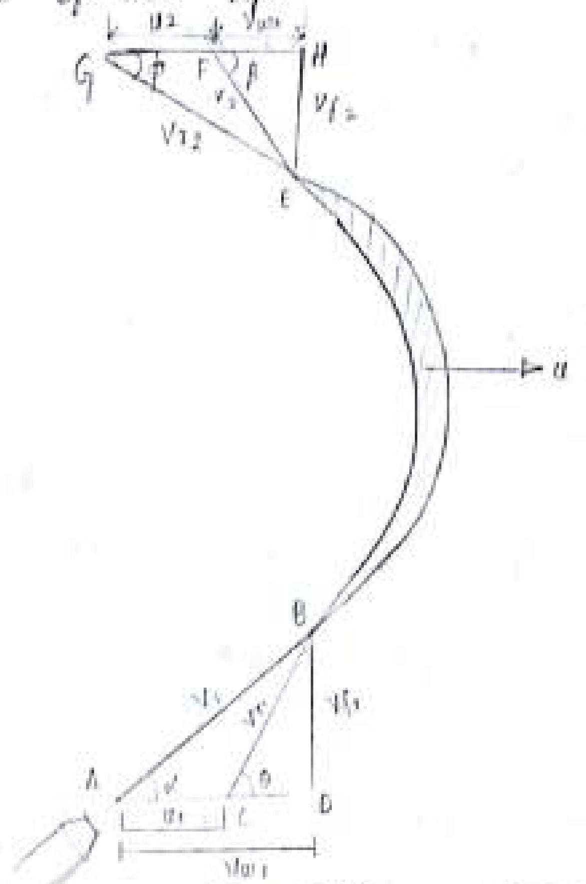
Work done per sec per unit weight of flow $= F_x \times u$

$= 5.83 \times 20$

$= 116.58\, N\,m/s.$

$\therefore$ Power developed $= \frac{116.58}{1000} = 0.116\, kW.$

Force exerted by a jet of water on an unsymmetrical moving curved plate when jet strikes tangentially at one of the tips



Consider a jet of water striking a moving curved plate tangentially, at one of its tips. As plate is moving, the velocity with which jet of water strikes is equal to the relative velocity of the jet with respect to plate.

Let V_1 = velocity of the jet at inlet.

u_1 = velocity of the plate at inlet.

V_{r1} = relative velocity of jet and plate at inlet.

α = angle between the direction of the jet and direction of motion of the plate, also called guide blade angle

θ = angle made by the relative velocity with the direction of motion at inlet also called vane angle at inlet

V_{w1} = Velocity of whirl at inlet.
V_{f1} = velocity of flow at inlet
V_2 = velocity of jet at outlet.
u_2 = velocity of vane at outlet
β = angle b/n V_2 and direction of motion of vane at outlet
V_{r2} = relative velocity of the jet
ϕ = angle made by the relative velocity V_{r2} with the direction of motion also called vane angle at outlet
V_{w2} and V_{f2} = components of velocity V_2 in the direction of motion of vane and $\perp^r$ to the direction of vane
V_{w2} = Velocity of whirl at outlet
V_{f2} = velocity of flow at outlet.

The ΔABD and ΔEGH are called velocity triangles at inlet and outlet and are drawn as given below.

i. Velocity triangle at inlet : Take any point A and draw a line AB = V_1 in magnitude and direction which means line AB makes an angle α with the horizontal line AD. Next draw a line AC = u_1 in magnitude. Join C to B. Then CB represents the relative velocity of the jet at inlet. If the loss of energy is zero, then CB must be in the tangential direction to the vane at inlet. From B draw a vertical line BD in the downward direction to meet the horizontal line AC produced at D.

Then BD = represents the velocity of flow at inlet = V_{f1}

AD = " " " whirl at inlet = V_{w1}

$\hat{BCD}$ = vane angle at inlet = θ.

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Velocity triangle at Outlet : The water glides over the surface of the vane with a relative velocity equal to V_{r_1}, and will come out of the vane with relative velocity V_{r_1}.

$\therefore V_{r_2} = V_{r_1}$.

Draw EG in the tangential direction of the vane at outlet and $EG = V_{r_2}$ from G. draw a line GF in the direction of vane at outlet and equal to u_2, the velocity of the vane at outlet. Join EF Then EF represents the absolute velocity of the jet at outlet in magnitude and direction from E draw a vertical line EH to meet the line GF produced at H. Then

EH = velocity of flow at outlet $= V_{f_2}$

FH = whirl at outlet $= V_{w_2}$

ϕ = angle of vane at outlet.

β = angle made by V_2 with the direction of motion of vane at outlet

And $u_1 = u_2 = u$ = velocity of vane in the direction of motion, $V_{r_1} = V_{r_2}$

Force exerted by the jet in the direction of motion $= \rho a V_{r_1}$ [initial velocity - final velocity]

$$= \rho a V_{r_1}\left[(V_{r_1}\cos\theta) - (-V_{r_2}\cos\phi)\right]$$

$$= \rho a V_{r_1}\left[(V_{w_1} - u_1) - (-(u_2 + V_{w_2}))\right]$$

$$= \rho a V_{r_1}\left[V_{w_1} + V_{w_2}\right]$$

The above equation is true only if β is an acute angle.

If $\beta = 90^\circ$ $\quad F_x = \rho a V_{r_1}\left[V_{w_1}\right]$

if β is obtuse $\quad F_x = \rho a V_{r_1}\left[V_{w_1} - V_{w_2}\right]$

Thus in general $\quad F_x = \rho a V_{r_1}\left[V_{w_1} \pm V_{w_2}\right]$

Work done per sec on the vane by the jet = force × distance per sec in the direction of force.

$$= F_x \times u$$

$$= \rho a V_{r1} [V_{w1} \pm V_{w2}] \times u$$

Work done per sec per unit weight of fluid striking per sec

$$= \frac{\rho a V_{r1} [V_{w1} \pm V_{w2}] \times u}{\text{weight of fluid/s}} \quad \frac{Nm/s}{N/s}$$

$$= \frac{\rho a V_{r1} [V_{w1} \pm V_{w2}] \times u}{\rho a V_{r1} \times g} \quad Nm/N$$

$$= \frac{1}{g} [V_{w1} \pm V_{w2}] \times u$$

Work done per sec per unit ~~weight~~ mass of fluid per sec

$$= \frac{\rho a V_{r1} [V_{w1} \pm V_{w2}] \times u}{\text{mass of fluid/s}}$$

$$= \frac{\rho a V_{r1} [V_{w1} \pm V_{w2}] \times u}{\rho a V_{r1}}$$

$$= [V_{w1} \pm V_{w2}] \times u \quad Nm/kg$$

Efficiency of the jet

$$\eta = \frac{\rho a V_{r1} (V_{w1} \pm V_{w2}) \times u}{\frac{1}{2} (\rho a V_1) V_1^2}$$

A jet of water having a velocity of 20 m/s strikes a curved vane, which is moving with a velocity of 10 m/s. The jet makes an angle of 20° with the direction of motion of vane at inlet and leaves at an angle of 130° to the direction of motion of vane at outlet. Calculate (i) Vane angles (ii) Work done per sec per unit weight of water striking the vane per second

Soln:

$V_1 = 20\,m/s$

$u_1 = 10\,m/s$

$\alpha = 20°$

$\beta = 180 - 130 = 50°$

$u_1 = u_2 = u = 10\,m/s$

$Vr_1 = Vr_2$

(i) Vane angle i.e. θ and ϕ

In Δ^{le} BCD.

$$\tan\theta = \frac{BD}{CD}$$

$$= \frac{Vf_1}{(Vw_1 - u_1)}$$

In Δ^{le} ABD.

$$\sin\alpha = \frac{Vf_1}{V_1}$$

$Vf_1 = \sin\alpha \; V_1$

$= \sin 20 \times 20$

$= \underline{6.84\,m/s}$

$$\cos\alpha = \frac{Vw_1}{V_1}$$

$Vw_1 = \cos\alpha \; V_1$

$= \cos 20 \times 20$

$= 18.794\,m/s.$

$$\therefore \tan\theta = \frac{6.84}{(18.794 - 10)}$$

$$\theta = \tan^{-1}\left[\frac{6.84}{8.79}\right]$$

$$\boxed{\theta = 37°50.5}$$

From Δ^{le} BCD

$\sin\theta = \frac{V_{f1}}{V_{r1}}$

$\sin 37°36'4'' = \frac{6.80}{V_{r1}}$

$\boxed{V_{r1} = 11.19 \text{ m/s}}$

$V_{r1} = V_{r2} = 11.19$ m/s

From Δ^{le} EFG

Applying sine rule

$$\frac{V_{r2}}{\sin(180-\beta)} = \frac{u_2}{\sin(\beta-\phi)}$$

$$\frac{11.19}{\sin(180-50)} = \frac{10}{\sin(50-\phi)}$$

$$\boxed{\phi = 6°33'6''}$$

ii) Work done per second per unit weight of water striking the vane per sec

$$= \frac{\rho a V_{r1}[V_{w1}+V_{w2}]u}{\rho a V_{r1} \times g}$$

$$\cos\phi = \frac{V_{w2}+u_2}{V_{r2}}$$

$$= \frac{[8.779 + 1.061]10}{9.81}$$

$11.19(\cos 6°33'6'') = V_{w2}+10$

$V_{w2} = 1.061$ m/s

$= \underline{20.04}$ Nm/N

→ A jet of water having a velocity of 40m/s strikes a curved vane which is moving with a velocity of 20m/s. The jet makes an angle of 30° with the direction of motion of vane at inlet and leaves at an angle of 90° to the direction of motion of vane at outlet. Draw velocity triangles at inlet and outlet and determine the vane angles at inlet and outlet.

Soln:

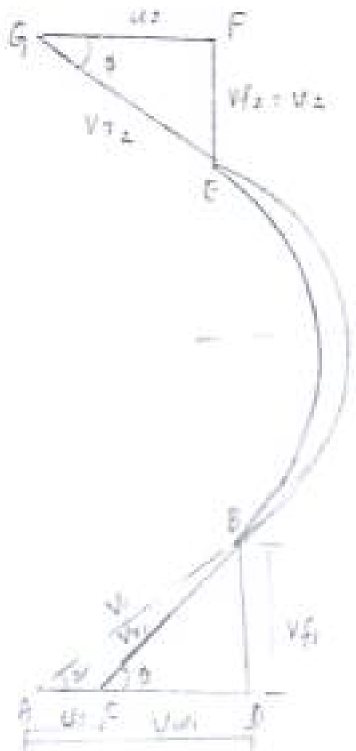
$V_1 = 40 \, m/s$

(i)

$u_1 = u_2 = u = 20 \, m/s$

$\alpha = 30°$

$\beta = 90°$



In Δ^{le} ABD

$\sin\alpha = \frac{V_{f1}}{V_1}$

$\sin 30° \times 40 = V_{f1}$

$= 20 \, m/s$

$\cos\alpha = \frac{V_{w1}}{V_1}$

$\cos 30° = \frac{V_{w1}}{40}$

$V_{w1} = 40 \times \cos 30°$

$= \underline{34.64 \, m/s}$

$CD = V_{w1} - u_1$

$= 34.64 - 20$

$= \underline{14.64 \, m/s}$

In Δ^{le} BCD

$\tan\theta = \frac{20}{14.64}$

$\underline{\theta = 53°}$

$\cos\theta = \frac{14.64}{V_{r1}}$

$V_{r1} = 24.78 \, m/s$

(ii) At outlet

$\Delta GEF \Rightarrow \cos\phi = \frac{u_2}{V_{r2}}$

$\cos\phi = \frac{20}{24.78}$

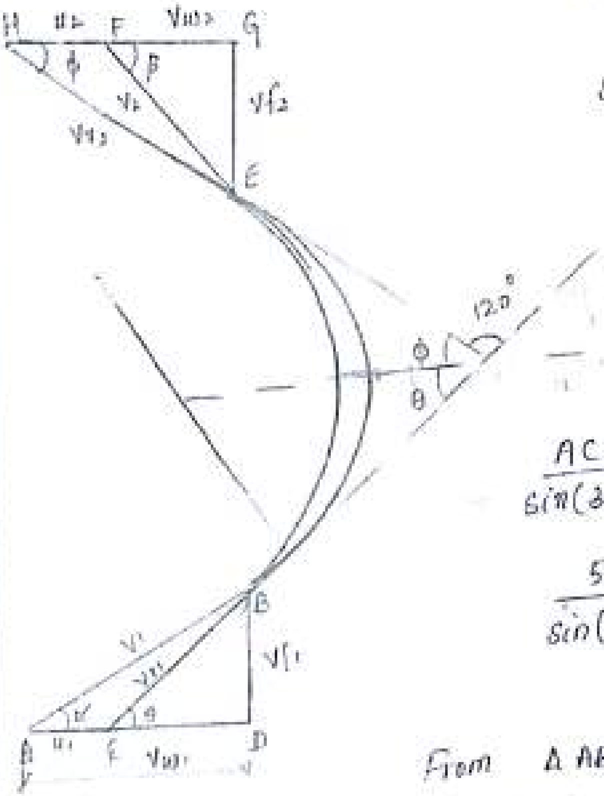
$\cos\phi = 0.8$

$\phi = \underline{\underline{36°10'}}$

A jet of water having a velocity of 15 m/s strikes a curved vane which is moving with a velocity of 5 m/s. The vane is symmetrical and is so shaped that the jet is deflected through 120°. Find the angle of the jet at inlet of the vane so that there is no shock. What is the absolute velocity of the jet at outlet in magnitude and direction and the work done per unit weight of water.

Soln :- $V_1 = 15\ m/s$

$u = 5\ m/s$



Since vane is symmetrical $\theta = \phi$

$\phi + \phi + 120^\circ = 180^\circ$

$\theta + \phi = 60^\circ$

$\theta = \phi = 30^\circ$

Applying sine rule for ΔACB

$$\frac{AC}{\sin(30-\alpha)} = \frac{AB}{\sin(180-\theta)}$$

$$\frac{5}{\sin(30-\alpha)} = \frac{15}{\sin 150}$$

$\alpha = 20°24'$

From ΔABD

$\cos\alpha = \frac{AD}{AB} \Rightarrow \cos 20°24' = \frac{V_{w1}}{V_1}$

$V_{w1} = 15 \times \cos 20°24'$

$= \underline{14.05\ m/s}$

$\sin\alpha = \frac{V_{f1}}{V_1} \Rightarrow \sin 20°24' \times 15 = V_{f1}$

In $\Delta^{le} BCD$

$\sin 30° = \frac{V_{f1}}{V_{r1}} \Rightarrow V_{r1} = 10.46\ m/s$

$V_{r1} = V_{r2} = 10.46\ m/s$

From Δ^{le} EHG

$\sin\phi = \frac{V_{f2}}{V_{r2}}$

$\sin 30° \times 10.46 = V_{f2}$

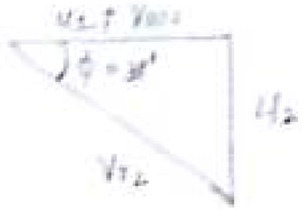
$V_{f2} = 5.23\ m/s$

$\cos\phi = \frac{u_2 + V_{w2}}{V_{r2}}$

$10.46 \cos 30° - 5 = V_{w2}$

$V_{w2} = 4.06\ m/s$

$V_{r1} = V_{r2} = 10.46\ m/s$



From Δ^{le} EHG

$$\cos\phi = \frac{u_2 + V_{w2}}{V_{r2}}$$

$(\cos 30° \times V_{r2}) - u_2 = V_{w2}$

$V_{w2} = 4.05\ m/s$

$$\sin\phi = \frac{V_{f2}}{V_{r2}}$$

$V_{f2} = 5.23\ m/s$

$$\tan\beta = \frac{V_{f2}}{V_{w2}} = \frac{5.23}{4.06} = 1.288$$

$\beta = 52°10.2'$

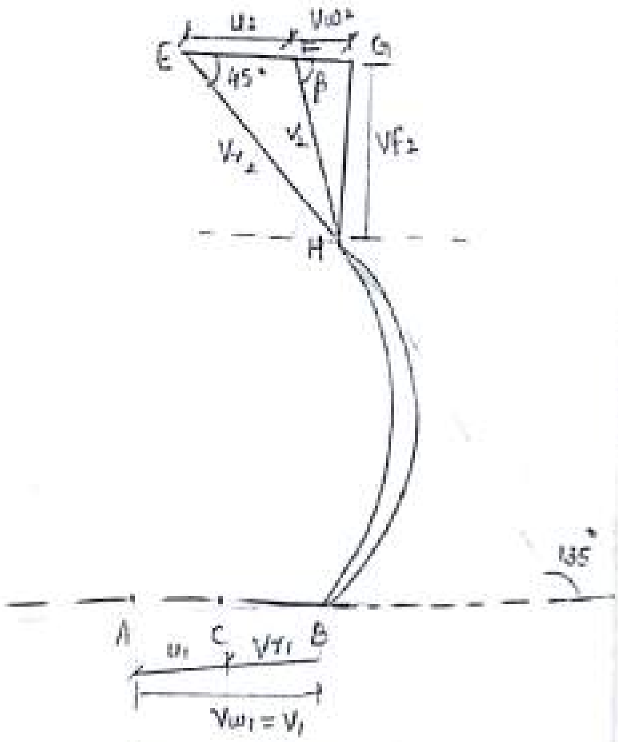
Work done per unit weight

$$= \frac{1}{g}\left[V_{w1} + V_{w2}\right] \times u \quad Nm$$

$$= \frac{1}{9.81}\left[V_1 \cos\alpha + 4.05\right] \times 5$$

$= 0.255\ Nm/N$

→ A jet of water having a velocity of 15 m/s, strikes a curved vane which is moving with a velocity of 5 m/s in the same direction as that of the jet at inlet. The vane is so shaped that the jet is deflected through 135°. The diameter of jet is 100mm. Assuming the vane to be smooth, find (i) force exerted by the jet on the vane in direction of motion. (ii) Power exerted on the vane. (iii) efficiency of the vane.



Soln: $V = 15\ m/s$

$u_1 = u_2 = u = 5\ m/s$

$\alpha = \theta = 0°$

$V_{r1} = V_1 - u_1$

$= 10\ m/s$

$V_{r1} = V_{r2} = 10\ m/s$

From Δ^{le} EGH

$\sin 45° = \dfrac{V_{f2}}{V_{r2}}$

$V_{f2} = \sin 45 \times V_{r2}$

$= \underline{7.07\ m/s}$

$\cos 45° = \dfrac{u_2 + V_{w2}}{V_{r2}}$

$(\cos 45° \times V_{r2}) - u_2 = V_{w2}$

$V_{w2} = 2.07\ m/s$

from Δ^{le} FGH

$\tan\beta = \dfrac{V_{f2}}{V_{w2}}$

$\beta = \underline{73°40'}$

i) force exerted by the jet on vane.

$F_x = \rho a V_{r1} [V_{w1} + V_{w2}]$

$= 1000 \times \pi \dfrac{(100 \times 10^{-3})^2}{4} [15 + 2.07]$

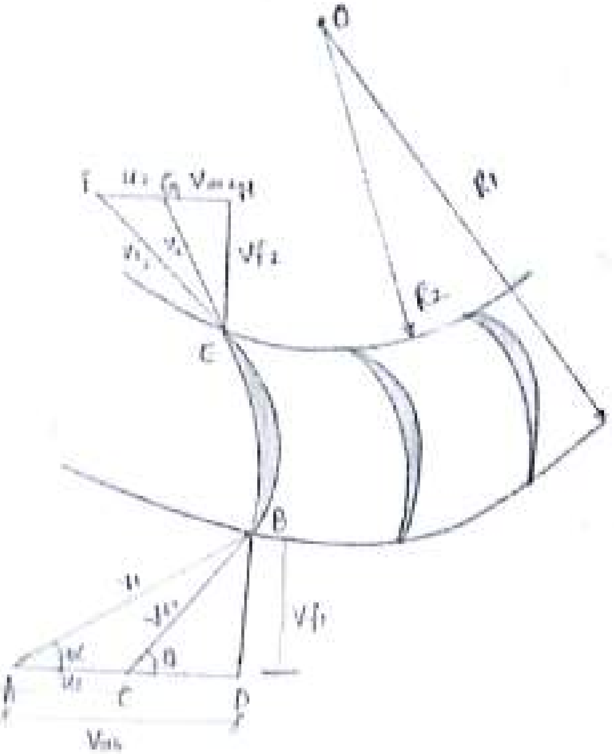
$= 1340.6\ N$

ii) Power $= \dfrac{F_x \times u}{1000} = 6.7\ kW$

iii) $\eta = \dfrac{1340.6 \times 5}{\frac{1}{2}\left(1000 \times \pi \frac{(100 \times 10^{-3})^2}{4} \times 15\right) 15^2} = 50.5\ \%$

Force exerted on a series of radial curved vanes

For a radial curved vane, the radius of the vane at inlet and outlet are differential. Therefore tangential velocities of radial vane at outlet and inlet will be different



R_1 = radius of wheel at inlet
R_2 = radius of wheel at outlet
ω = angular speed of the wheel
$u_1 = \omega R_1$ and $u_2 = \omega R_2$.
The velocity triangles are drawn as shown.
Force exerted by water on the wheel

$$T = \rho a V_1 \times V_{w_1} \times R_1 - (-\rho a V_1 \times V_{w_2} \times R_2)$$
$$= \rho a V_1 [V_{w_1} R_1 + V_{w_2} R_2]$$

Work done per sec = Force $\times$ angular velocity

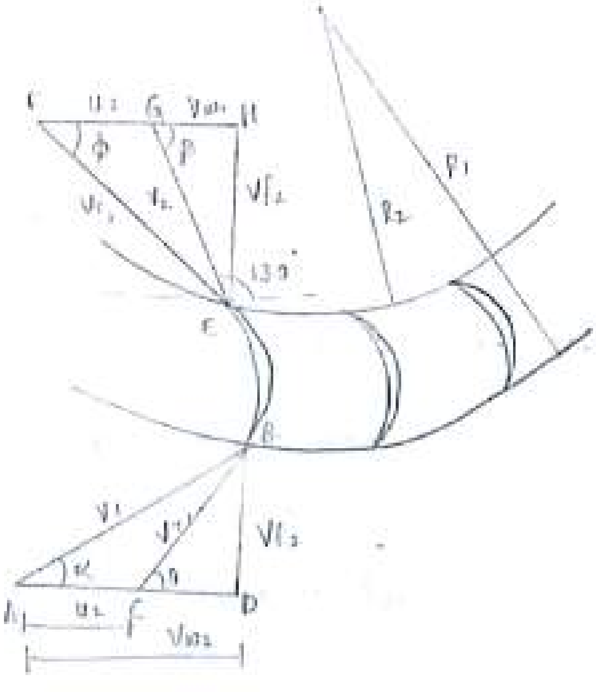
$$= \rho a V_1 [V_{w_1} u_1 + V_{w_2} u_2]$$

If β is obtuse
work done per sec
$$= \rho a V_1 [V_{w_1} u_1 - V_{w_2} u_2]$$

If $\beta = 90^\circ$ work done $= \rho a v_1 [V_{w_1} u_1]$.

Efficiency $\eta = \dfrac{2[V_{w_1}u_1 \pm V_{w_2}u_2]}{V_1^2}$

→ A jet of water having a velocity of 30 m/s strikes a series of radial curved vanes mounted on a wheel which is rotating at 200 rpm. The jet makes an angle of 20° with the tangent to the wheel at inlet and leaves the wheel with a velocity of 5 m/s at an angle of 130° to the tangent to the wheel at outlet. Water is flowing from outward in a radial direction. The outer and inner radii of the wheel are 0.5 m and 0.25 m respectively. Determine (i) Vane angles at inlet and outlet
(ii) Work done per unit weight of water
(iii) Efficiency of the wheel



Soln $V_1 = 30\text{ m/s}$

$\omega = \dfrac{2\pi N}{60} = \dfrac{2\times\pi\times 200}{60} = 20.94\text{ rad/s}$

$\alpha = 20^\circ$

$V_2 = 5\text{ m/s}$

$\beta = 180 - 130 = 50^\circ$

$R_1 = 0.5\text{ m}$

$R_2 = 0.25\text{ m}$

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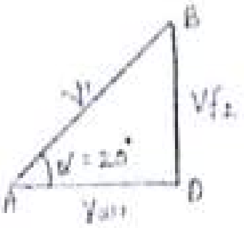
$u_1 = \omega R_1 = 20.94 \times 0.5 = 10.47\ m/s$

$u_2 = \omega R_2 = 20.94 \times 0.25 = 5.235\ m/s$

(ii) Vane angles

From Δ^{le} ABD



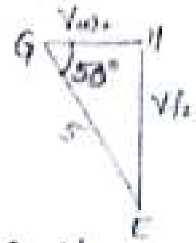
$\cos 20° = \frac{V_{w_1}}{V_1}$

$V_{w_1} = 28.19\ m/s$

$\sin 20° = \frac{V_{f_1}}{V_1}$

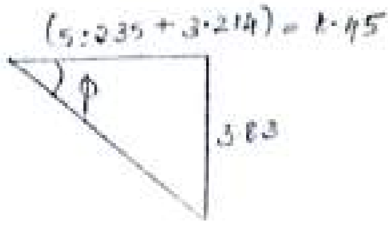
$V_{f_1} = 10.26\ m/s$.

From Δ^{le} EGH



$\cos 50° = \frac{V_{w_2}}{V_2} \Rightarrow V_{w_2} = 3.214\ m/s$

$\sin 50° = \frac{V_{f_2}}{V_2} \Rightarrow V_{f_2} = 3.83\ m/s$



$\tan\phi = \frac{3.83}{8.45}$

$\phi = \tan^{-1}(0.45)$

$\boxed{\phi = 24° 23.1'}$

Work done per sec

$= \rho a V_1 [V_{w_1} u_1 + V_{w_2} u_2]$

$\frac{\text{Work done per sec}}{\text{unit weight / sec}} = \frac{\rho a V_1 [V_{w_1} u_1 + V_{w_2} u_2]}{\rho a V_1 \times g}$

$= \left[\frac{28.19 \times 10.47 + 3.214 \times 5.235}{9.81}\right]$

$= \underline{31.8\ Nm/N}$

Efficiency $\eta = \frac{2[V_{w_1} u_1 + V_{w_2} u_2]}{V_1^2}$

$= \frac{2[28.19 \times 10.47 + 3.214 \times 5.235]}{30^2} \times 100 = 69.32\ \%$.

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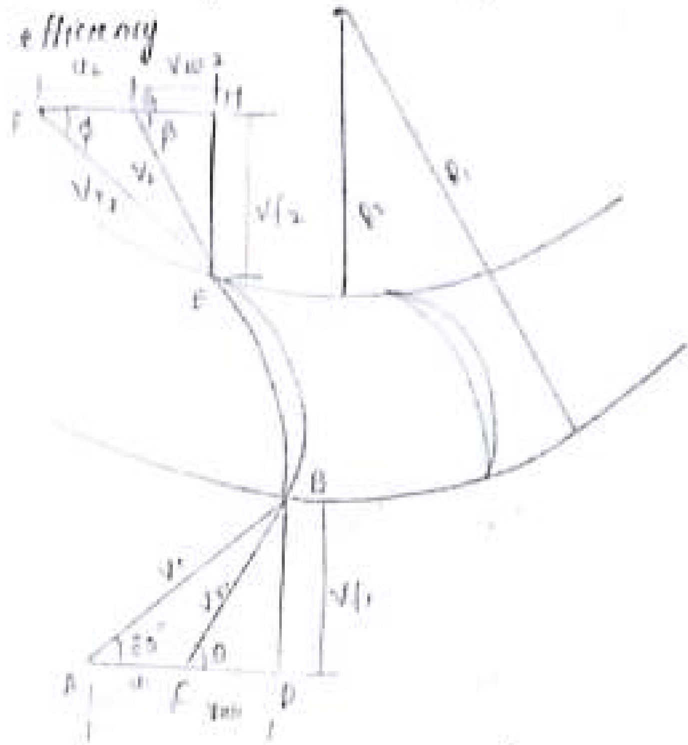
22) A jet of water having a velocity of 35 m/s impinges on a series of vanes moving with a velocity of 20 m/s. The jet makes an angle of 30° to the direction of motion of vanes when entering and leaves at an angle of 120°. Draw the velocity triangles and find

i) the vane angles
ii) the work done per unit weight of water
iii) the efficiency



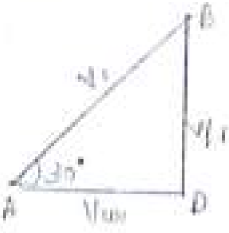
Soln: $V_1 = 35\ m/s$

$u_1 = u_2 = 20\ m/s$

$\alpha = 30°$

$\beta = 180° - 120° = 60°$

From Δ^{le} ABD



$\cos 30° = \frac{V_{w1}}{V_1}$

$V_{w1} = 35 \cos 30°$

$= 30.31\ m/s$

$\sin 30° = \frac{V_{f1}}{V_1}$

$V_{f1} = 35 \times \sin 30°$

$= 17.50\ m/s$

From

$V_{f1} = 17.50\ m/s$

$V_{w1} - u_1 = (30.31 - 20) = 10.31\ m/s$

$\tan\theta = \frac{V_{f1}}{(V_{w1} - u_1)}$

$= \frac{17.50}{10.31}$

$\phi = \tan^{-1}(1.69)$

$\phi = 60°$

$\sin\theta = \frac{V_{f1}}{V_{r1}}$

$V_{r1} = \frac{17.50}{\sin 60}$

$V_{r1} = 20.25\ m/s$.

From Δ^{le} EFG

$$\frac{u_2}{\sin(\beta-\phi)} = \frac{V_{r_2}}{\sin(180-\beta)}$$

$$\frac{20}{\sin(60-\phi)} = \frac{22.25}{\sin(180-60)}$$

$$\underline{\phi = 1°15'}$$

Work done per unit weight of water force

$$= \frac{\rho a V_{r_1}[V_{w_1} \pm V_{w_2}] \times u}{g \times \rho a V_{r_1}}$$

$$= 62.28 \ Nm/N.$$

$$\eta = \frac{2[V_{w_1} u_1 + V_{w_2} u_2]}{V_1^2}$$

$$= \underline{98.74\%}.$$

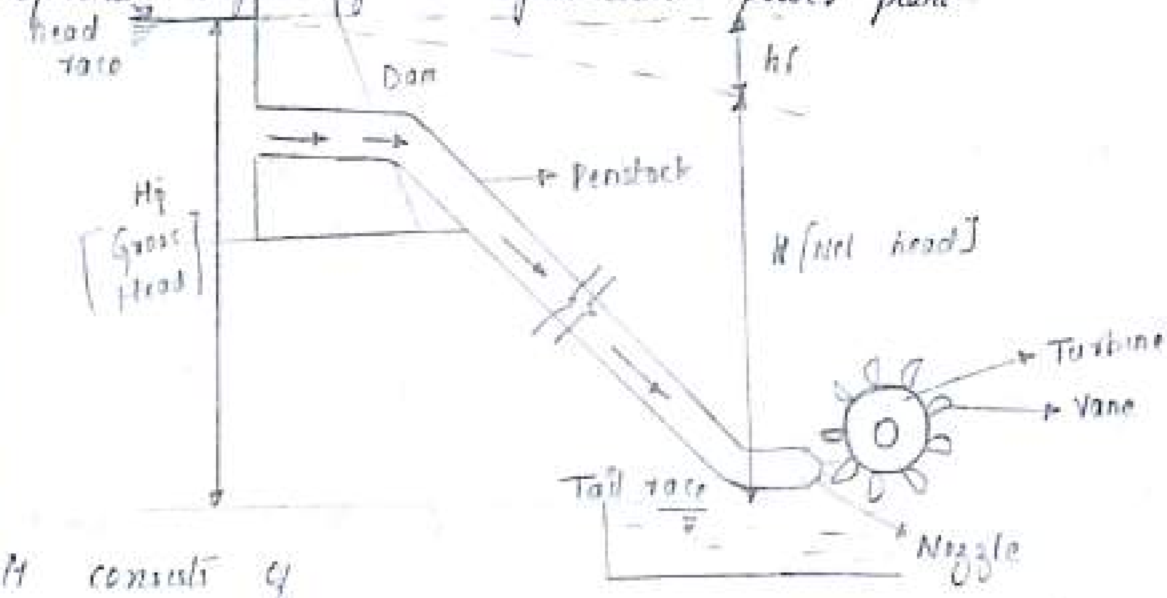
TURBINES

Hydraulic machines are defined as those machines which convert either hydraulic energy into mechanical energy or mechanical energy into hydraulic energy.

The hydraulic machines which converts the hydraulic energy into mechanical energy are called turbines.

The hydraulic machine which converts the ~~hy~~ mechanical energy into hydraulic energy are called Pumps

General Layout of a Hydroelectric power plant



It consists of

(1) A dam constructed across a river to store water.

(2) Pipes of large diameters called Penstocks which carry water under pressure from reservoir to turbines

(3) Turbines having different types of vanes fitted to the wheels

(4) Tail race which is a channel which carries water away from the turbine after the work has been done by the turbines. The surface of water in the tail race is also known as tail race

Heads in Turbines

Gross Head: The difference between the head race level and tail race level when no water is flowing is known as Gross Head denoted as 'H_g'

Net head: It is defined as the head available at the inlet of the turbine. It is also called the effective head. As the water flows from head race to tail race, loss of head due to friction between the water and penstock occurs. If 'h_f' is the head loss due to friction, the net head is given by

$$H = H_g - h_f$$

$$h_f = \frac{4fLV^2}{2gD}$$

Efficiencies of a Turbine

Hydraulic efficiency: It is defined as the ratio of power given by water to the runner of a turbine to the power supplied by the water at the inlet of the turbine.

$$\eta_h = \frac{\text{Power delivered to runner}}{\text{Power supplied at inlet}}$$

Mechanical efficiency: The ratio of the power available at the shaft of the turbine to the power delivered to the runner is defined as Mechanical efficiency.

$$\eta_m = \frac{\text{Power at the shaft of turbine}}{\text{Power delivered by water to the runner}}$$

Volumetric efficiency : The ratio of the volume of the water actually striking the runner to the volume of water supplied to the turbine is called as Volumetric efficiency

$$\eta_v = \frac{\text{Vol of water actually striking the runner}}{\text{Vol of water supplied to the turbine}}$$

Overall efficiency : It is defined as the ratio of power available at the shaft of the turbine to the power supplied by the water at the inlet of the turbine

$$\eta_o = \frac{\text{Vol available at the shaft of turbine}}{\text{Power supplied at the inlet of turbine}}$$

$$\eta_o = \eta_m \times \eta_h$$

Classification of turbines

1) Based on the type of energy at inlet

a) Impulse turbine : If the energy available at inlet is only kinetic energy then the turbine is known as an impulse turbine

b) Reaction turbine : If the energy available at inlet has both kinetic energy and pressure energy then the turbine is known as Reaction turbine

2) Based on the direction of flow through runner

a) Tangential flow turbine : If the water flows along the tangent of the runner then the turbine is a tangential flow turbine

b) Radial flow turbine : If the water flows in the radial direction through the runner then the turbine is called radial flow turbine

c) Axial flow turbine : If the water flows through the runner along the direction parallel to the axis of rotation of the runner, the turbine is called axial flow turbine

d) Mixed flow turbine : If the water flows through the runner in the radial direction but leaves in the direction parallel to axis of rotation of the runner, the turbine is called Mixed flow turbine.

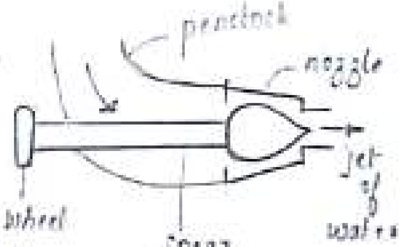
c) Based on the head at the inlet of turbine
  a) High head turbine
  b) Medium head turbine
  c) Low head turbine

4) Based on the specific speed of turbine
  a) Low specific speed turbine
  b) Medium specific speed turbine
  c) High specific speed turbine

PELTON WHEEL

It is a tangential flow, impulse turbine
The main parts of the Pelton turbine are

a) Nozzle and flow regulating arrangement



The amount of water striking the buckets of the runner is controlled by providing a spear in the nozzle as shown. The spear is a conical needle which is operated either by a hand wheel or automatically. When the spear is pushed forward into the nozzle the amount of water striking the runner is reduced.

b) Runner with buckets

It consists of a circular disc on the periphery of which a number of evenly spaced buckets are fixed. The shape of the bucket is of a double hemispherical cup or bowl. Each bucket is divided into 2 symmetrical parts by a dividing wall known as splitter. The jet of water strikes the splitter and divides the jet into 2 equal parts and comes out at the outer edge of the bucket

The buckets are shaped in such a way that the jet gets deflected through 160° or 170°.

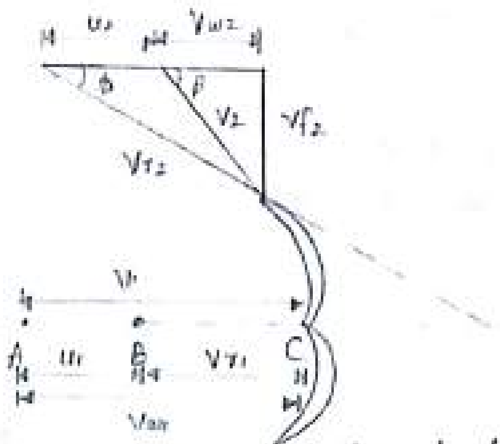
c) Casing.

The function of the casing is to prevent the splashing of water and to discharge water to tail race. It doesnot perform any hydraulic function.

d) Breaking jet

When the nozzle is completely closed by moving the spear in the forward direction, the amount of water striking the runner reduces to zero But due to inertia, the runner goes on revolving for a long time To stop the runner in a short time, a small nozzle is provided which directs the jet on the back of vanes This jet of water is called Breaking jet.

WORK DONE FOR PELTON WHEEL



The jet of water from the nozzle strikes the bucket at the splitter, which splits up the jet into two parts. These parts of the jet, glides over the inner surfaces and comes out at the outer edge The splitter is the inlet tip and outer edge of the bucket is the outlet tip of the bucket. The inlet velocity triangles are drawn at splitter and outlet velocity triangles are drawn at outlet tip of the bucket.

Net head = Gross head − h_f

$= H_g - h_f$

velocity of jet at inlet $V_1 = \sqrt{2gH}$

$u_1 = u_2 = u = \frac{\pi D N}{60}$

At inlet triangle $V_{r1} = V_1 - u_1$

$V_{w1} = V_1$

$\alpha = 0^\circ$ and $\theta = 0^\circ$

At outlet triangle $V_{r2} = V_{r1}$

$V_{w2} = V_{r2}\cos\phi - u_2$

The force exerted by the jet of water in x direction

$F_x = \rho a V_1 [V_{w1} + V_{w2}]$ N +ve for acute β angle

Work done

$F_x \times u = \rho a V_1 [V_{w1} + V_{w2}] \times u$ Nm/s

Power $= \frac{\rho a V_1 [V_{w1} + V_{w2}] \times u}{1000}$ kW

Work done/s per unit weight of water striking/s $= \frac{\rho a V_1 [V_{w1} + V_{w2}] u}{\rho a V_1 \times g}$

$= \frac{1}{g} [V_{w1} + V_{w2}] u$

Efficiency $\eta = \frac{\rho a V_1 [V_{w1} + V_{w2}] \times u}{\frac{1}{2}(\rho a V_1) V_1^2}$

$= \frac{2[V_{w1} + V_{w2}] \times u}{V_1^2}$ $V_{w2} = V_{r2}\cos\phi - u_2$
$= (V_1 - u)\cos\phi - u$

$= \frac{2[V_1 + (V_1 - u)\cos\phi - u] \times u}{V_1^2}$

$= \frac{2(V_1 - u)[1 + \cos\phi] u}{V_1^2}$

For maximum efficiency

$$\frac{d}{d(u)}(\eta) = 0$$

$$\frac{d}{du}\left[\frac{2u(V_1 - u)(1+\cos\phi)}{V_1^2}\right] = 0$$

$$\frac{(1+\cos\phi)}{V_1^2}\frac{d}{du}(2uV_1 - 2u^2) = 0$$

$$\frac{1+\cos\phi}{V_1^2} \neq 0 \qquad 2V_1 - 4u = 0$$

$$\boxed{u = \frac{V_1}{2}}$$

substituting u in eqn *

$$\eta = \frac{2\left[V_1 - \frac{V_1}{2}\right]\left[1+\cos\phi\right] \times \frac{V_1}{2}}{V_1^2}$$

$$= \frac{2\frac{V_1}{2}(1+\cos\phi) \times \frac{V_1}{2}}{V_1^2}$$

$$\boxed{\eta = \frac{(1+\cos\phi)}{2}}$$

Note :

i> the velocity of the jet at inlet

$$V_1 = C\sqrt{2gH}$$

$C_v = 0.95$ or 0.99

H = Net head on turbine

ii> The velocity of wheel (u) is given by

$$u = \phi\sqrt{2gH}$$

ϕ = speed ratio = 0.43 to 0.48

iii> The deflection angle is 165°, if not given

iv> The mean diameter or the pitch diameter D is given by $u = \frac{\pi D N}{60}$

v> Jet Ratio : it is defined as the ratio of pitch diameter (D) of the Pelton wheel to the dia of the jet

$$m = \frac{D}{d} \quad (= 12 \text{ in most cases})$$

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Number of buckets is given by

$$Z = 15 + \frac{D}{2d} = 15 + 0.5(m)$$

m - Jet ratio

Number of jets. N is obtained by dividing the total rate of flow through the turbine by the rate of flow of water through a single jet

→ A Pelton wheel has a mean bucket speed of 10m/s with a jet of water flowing at the rate of 700lps under a head of 30m The buckets deflect the jet through an angle of 160° Calculate the power given by water to the runner and the hydraulic efficiency of the turbine Assume $C_v = 0.98$.

Soln

$u = 10m/s$

$Q = 700 \times 10^{-3}\ m^3/s$

$H = 30m$

$\alpha = 180 - 160°$

$= 20°$

$C_v = 0.98$

$V_1 = C_v\sqrt{2gH}$

$= 0.98 \times \sqrt{(2 \times 9.81 \times 30)}$

$= 23.77\ m/s$

$V_{r1} = V_1 - u_1$

$= 23.77 - 10$

$= 13.77\ m/s$

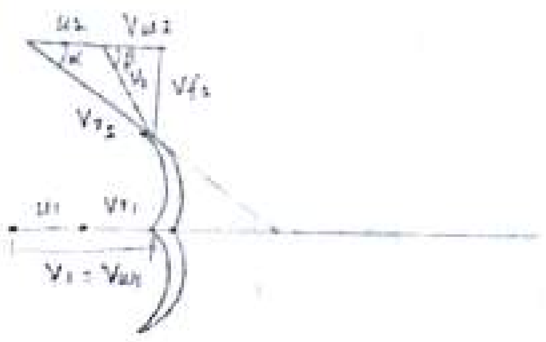
$V_{w1} = V_1 = 23.77\ m/s$

$V_{r2} = V_{r1} = 13.77\ m/s$

$V_{w2} = V_{r2}\cos\phi - u_2$

$= 13.77\cos 20° - 10$

$= 2.94\ m/s$



Work done /s $= \rho a V_1[V_{w1} + V_{w2}] \times u$

$= 1000 \times 0.7 \times [23.77 + 2.94] \times 10$

$= 186970\ Nm/s$.

$$\text{Power} = \frac{186770}{1000} = 186.77\,kW$$

$$\text{Efficiency} \quad \eta = \frac{2[V_{w1} + V_{w2}] \times u}{V_1^2}$$

$$= \frac{2[23.77 + 2.94] \times 10}{23.77 \times 23.77}$$

$$= \underline{94.54\,\%}$$

→ A Pelton wheel is to be designed for the foll specification: Shaft power = 11772 kW, Head = 380 m, Speed = 750 rpm, Overall efficiency = 86%, jet diameter is not to exceed to one sixth of the wheel diameter. Determine (i) the wheel diameter (ii) The number of jets required (iii) diameter of jet

$C_v = 0.985$ and $\phi = 0.45$

__Soln:__

$SP = 11772\,kW$

$H = 380\,m$

$N = 750\,rpm$

$\eta_o = 86\%$ or 0.86

$d = \frac{1}{6} \times D$

$\frac{d}{D} = \frac{1}{6}$

$C_v = 0.985$

$\phi = 0.45$

velocity of jet $V_1 = C_v\sqrt{2gH}$

$= 0.985\sqrt{2 \times 9.81 \times 380}$

$= 85.05\,m/s$

$u = \phi\sqrt{2gH}$

$= 0.45\sqrt{2 \times 9.81 \times 380}$

$= 38.85\,m/s$

$u = \frac{\pi D N}{60}$

$\frac{38.85 \times 60}{\pi \times 750} = D$

$D = 0.989$

$d = \frac{1}{6} \times D$

$= \frac{1}{6} \times 0.989$

$= 0.165\,m$

Discharge of one jet q = area of jet × velocity of jet

$$= \frac{\pi}{4}(0.165)^2 \times 85.05$$

$$= 1.818 \, m^3/s$$

$$\eta_o = \frac{S.P}{W.P} = \frac{11772}{\frac{\rho \times g \times Q \times H}{1000}}$$

$$0.86 = \frac{11772 \times 1000}{1000 \times 9.81 \times Q \times 380}$$

$$Q = 3.672 \, m^3/s$$

No of jets = $\frac{\text{Total discharge}}{\text{Discharge of one jet}} = \frac{Q}{q}$

$$= \frac{3.672}{1.818} = 2 \text{ jets}$$

→ The water available for a Pelton wheel is $4 \, m^3/s$ and the total head from the reservoir to the nozzle is 250 m The turbine has 2 runners with 2 jets per runner All the four jet have the same diameter. The pipeline is 3000 m long. The efficiency of power transmission through the pipe line and the nozzle is 91% and efficiency of each runner is 90%. The velocity co-efficient of each nozzle is 0.975 and co-efficient of friction '4f' for the pipe is 0.0045. Determine

1) The power developed by the turbine
2) The dia of the jet
3) The dia of the pipeline

Soln: $Q = 4 \, m^3/s$

$H_g = 250 m$

$L = 3000 m$

$\eta = 91\% = 0.91$

$C_v = 0.975$

$4f = 0.045$

$$\eta \text{ of Power transmission} = \frac{H_g - h_f}{H_g}$$

$$0.91 = \frac{250 - hf}{250}$$

$$hf = 22.5m$$

Net head $= H_g - hf$

$$H = 250 - 22.5 = 221.5m.$$

$$V_1 = C_v\sqrt{2gH}$$

$$= 0.995\sqrt{2 \times 9.81 \times 221.5}$$

$$= 65.14 \ m/s.$$

$$\eta_h = \frac{\text{Power developed by turbine}}{\text{Water power}}$$

$$\text{Water power} = \frac{1}{2}(m V_1^2)/1000$$

$$= \frac{1}{2}(\rho \times Q) V_1^2/1000$$

$$= \frac{\frac{1}{2} \times 1000 \times 65.14^2 \times 4}{1000}$$

$$= \underline{8486.44 kW}.$$

$$0.9 \times 8486.44 = \text{Power developed}$$

$$\text{Power developed} = 7637.8 kW.$$

$$\text{No of jets} = \frac{\text{Total discharge}}{\text{discharge through each jet}}$$

$$4 = \frac{4}{\text{discharge through each jet}}$$

$$q = 1 \ m^3/s$$

$$hf = \frac{4fLV^2}{2gD} = \frac{0.0045 \times 3000 \times V^2 \cdot 4}{2gD}$$

$$22.50 = \frac{0.0045 \times 3000 \times \left[\frac{4Q}{\pi D^2}\right]^2}{2gD}$$

$$D^5 = 0.7963$$

$$\boxed{D = 0.955m}$$

Design of a Pelton wheel

It includes finding out the below specifications

1) Diameter of the jet (d)

2) Diameter of the wheel (D)

3) Width of the bucket = 5×d

4) Depth of the bucket = 1.2×d

5) Number of buckets on the wheel

$$Z = 15 + \frac{D}{2d}$$

→ A Pelton wheel is to be designed for a head of 60m when running at 200 rpm. The Pelton wheel develops 95.6475 kW shaft power. The velocity of the buckets = 0.45 times the velocity of the jet. Overall efficiency = 0.85 and C_v = 0.98.

Soln :

$H = 60m$

$N = 200rpm$

$S.P = 95.6475 kW$

$u = 0.45 \times v_1$

$\eta_o = 0.85$

$C_v = 0.98$

$$V = C_v\sqrt{2gH}$$

$$= 0.98 \times \sqrt{(2 \times 9.81 \times 60)}$$

$$= 33.62 \text{ m/s}$$

$$u = \frac{\pi DN}{60}$$

$$u = 0.45 \times V = 0.45 \times 33.62$$

$$= 15.13 \text{ m/s}$$

$$15.13 = \frac{\pi D \times 200}{60}$$

$$D = \underline{1.44m}$$

$$\eta_o = \frac{S.P}{W.P}$$

$$\text{Water Power} = \frac{\rho \times Q \times g \times H}{1000}$$

$$= \frac{1000 \times Q \times 9.81 \times 60}{1000}$$

$$0.85 = \frac{95.6475 \times 1000}{1000 \times Q \times 9.81 \times 60}$$

$$Q = 0.1912 \text{ m}^3/\text{s}$$

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Discharge = area of jet × velocity

$0.1912 = \frac{\pi d^2}{4} \times 33.62$

$d = 0.085m$

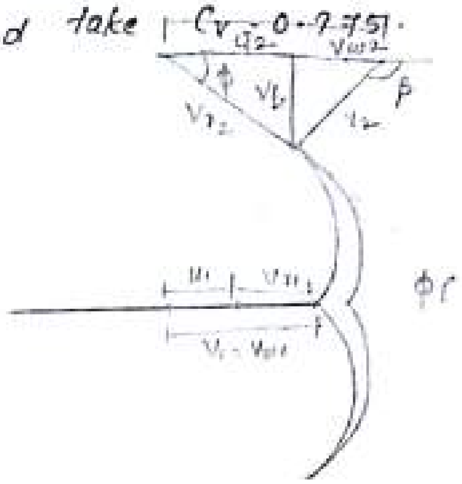
Width of bucket = $5 \times d = 5 \times 0.085 = 0.4m$

Depth of bucket = $1.2 \times d = 1.2 \times 0.085 = 0.102m$.

No of buckets $z = 15 + \frac{D}{2d} = 15 + \frac{1.44}{2 \times 0.085}$

$= 23.5 \simeq 24$

→ Determine the power given by the jet of water to the runner of a Pelton wheel which is having tangential velocity as 20m/s. The net head on the turbine is 50m and discharge through the jet water is $0.03 m^3/s$. The side clearance angle is 15° and take Cv = 0.975.



Soln :

$u = u_1 = 20 m/s$.

$H = 50m$

$Q = 0.03 m^3/s$

$Cv = 0.975$

$\phi = 15°$

$V_1 = Cv\sqrt{2gH}$

$= 0.975 \times \sqrt{2 \times 9.81 \times 50}$

$= 30.54 m/s$.

$V_{w1} = V_1 = 30.54 m/s$.

$V_{r1} = V_1 - u_1$

$= 10.54 m/s$.

$V_{r1} = V_{r2} = 10.54 m/s$.

$V_{r2}\cos\phi = 10.54 \times \cos 15°$

$= 10.18 m/s$

$V_{w2} = u_2 - V_{r2}\cos\phi$

$= 20 - 10.18 = 9.82 m/s$

β is an obtuse angle, the work done per sec

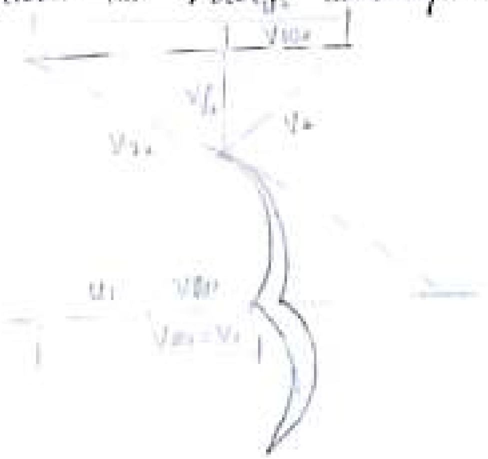
$= \rho a V_1 [V_{w_1} - V_{w_2}] \times u$

$= 1000 \times 0.03 \times [30.54 - 9.82] \times 20$

$= 12432\ Nm/s$

Power $= \frac{12432}{1000} = 12.43\ kW$

→ A 137 mm diameter jet of water issuing from a nozzle impinges on the buckets of a Pelton wheel and the jet is deflected through an angle of 165° by the buckets. the head available at the nozzle is 400m Assuming $C_v = 0.97$ and $\phi = 0.46$ and reduction in relative velocity while passing through buckets as 15%. Find (i) the force exerted by the jet on buckets (ii) Power developed



$d = 137 \times 10^{-3}\ m$

$\phi = 15°$

$H = 400 m$

$C_v = 0.97$

$\phi = 0.46$

$V_{r_2} = 0.85 V_{r_1}$

$V_1 = C_v \sqrt{2gH}$

$= 0.97 \times \sqrt{2 \times 9.81 \times 400}$

$= 85.93\ m/s$

$u = \phi \sqrt{2gH}$

$= 0.46 \sqrt{2 \times 9.81 \times 400}$

$= 40.75\ m/s$

$V_{r_1} = V_1 - u_1 = 45.18\ m/s$

$V_{r_2} = 0.85 V_{r_1} = 0.85 \times 45.18 = 38.40\ m/s$

$V_{r_2} \cos\phi = 38.40 \times \cos 15 = 37.092$

$V_{w_2} = u_2 - V_{r_2} \cos\phi$

$= 40.75 - 37.092$

$= 3.658\ m/s$

Force exerted by jet on buckets

$$F_x = \rho a V_1 [V_{w1} - V_{w2}]$$

$$F_x = 1000 \times 0.01474 \times 85.93 (85.93 - 3.658)$$

$$= \underline{104006\ N}$$

$$\text{Power} = \frac{F_x \times u}{1000} = \underline{4206.4\ kW}$$

→ The three jet Pelton turbine is required to generate 10,000 kW under a net head of 400 m. The blade angle at outlet is 15° and the reduction in the relative velocity while passing over the blade is 5%. If the overall efficiency of the wheel is 80%, $C_v = 0.98$ and speed ratio = 0.46, then find (i) the dia of the jet (ii) total flow in m³/s (iii) force exerted by the jet

Soln

$x = 3$

$S.P = 10{,}000\ kW$

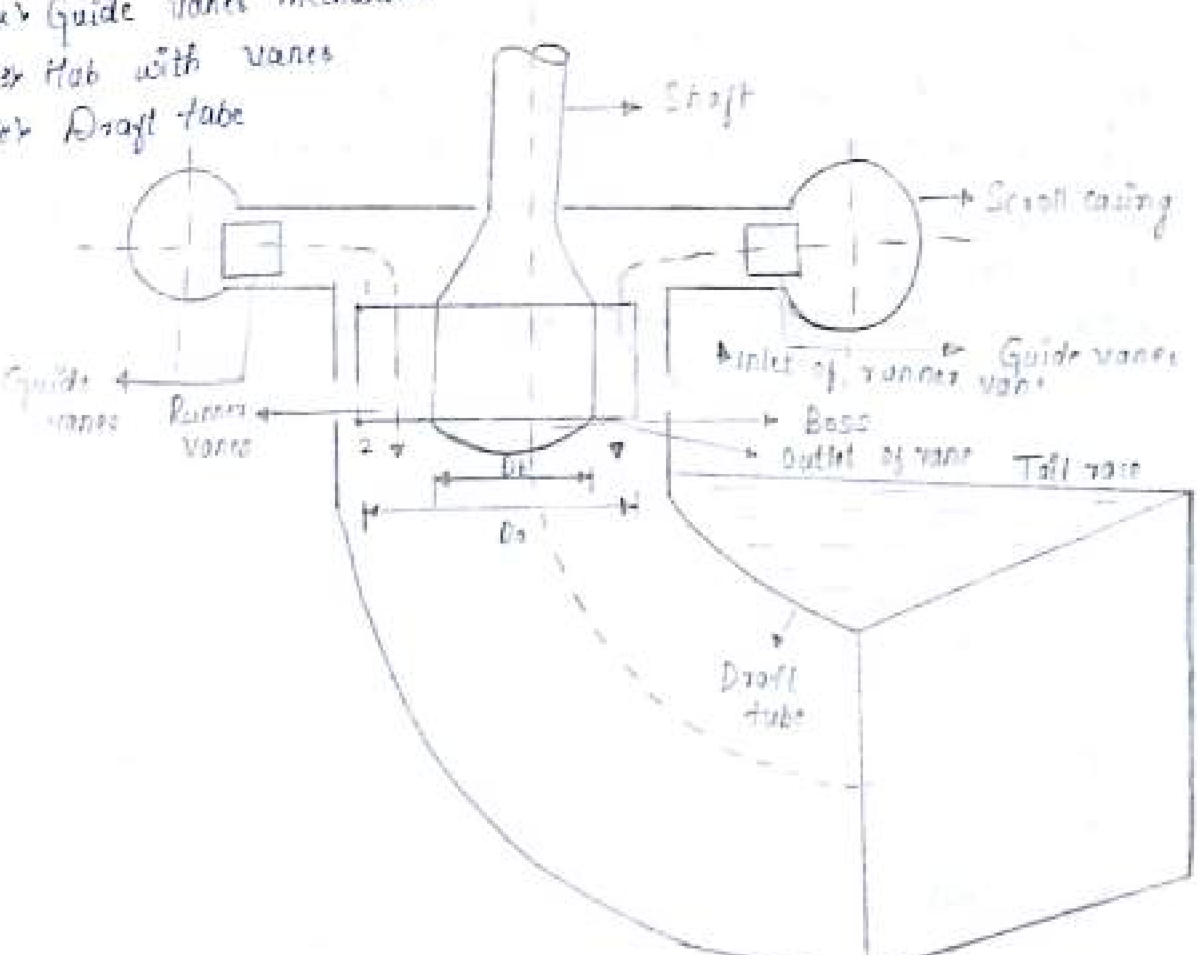
KAPLAN TURBINE

If the water flows parallel to the axis of rotation of the shaft, the turbine is called as axial flow turbine. If the head at the inlet of the turbine is the sum of pressure energy and kinetic energy, then it is called reaction turbine.

For axial flow reaction turbine, the shaft is vertical and the lower end of the shaft is larger and it is called 'Hub' or 'Boss'. The vanes are fixed to this hub.
If the vanes are fixed and are not adjustable, the turbine is known as Propeller turbine.
If the vanes are adjustable, then the turbine is known as Kaplan turbine and is used for large quantity and low head.

Main Parts of a Turbine.

(1) Scroll casing
(2) Guide vanes mechanism
(3) Hub with vanes
(4) Draft tube



The discharge through the runner is obtained by

$$Q = \frac{\pi}{4}(D_o^2 - D_b^2) \times V_{f1}$$

D_o : outer diameter of the runner

D_b : Diameter of the hub / boss

V_{f1} : velocity of flow at inlet

Points to be remembered

1) $u_1 = u_2 = \frac{\pi D_o N}{60}$ D_o : outer dia of runner

2) Velocity of flow at inlet and outlet are equal

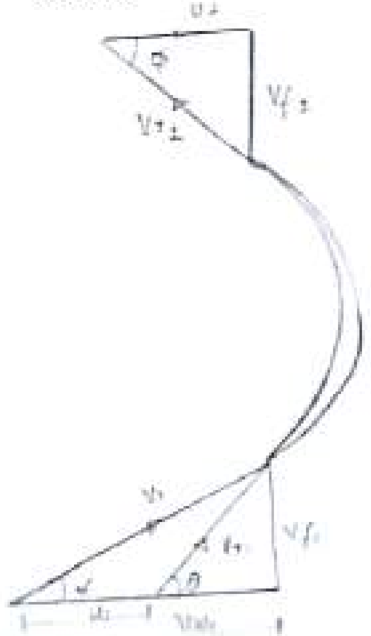
$$V_{f1} = V_{f2}$$

3) Area of flow at inlet = Area of flow at outlet

$$= \frac{\pi}{4}(D_o^2 - D_b^2)$$

4) $\eta_h = \frac{V_{w1} u_1}{gH}$

→ A Kaplan Turbine working under a head of 20m develops 11772kW shaft power. The outer diameter of the runner is 3.5m and hub diameter is 1.75m. The guide blade angle at the extreme edge of the runner is 35°. The hydraulic and overall efficiencies of the turbines are 88% and 84% respectively. If the velocity of the whirl is zero at outlet, determine (i) Runner vane angles at inlet and outlet at the extreme edge of the runner
(ii) Speed of the turbine



Soln :

$H = 20m$

$SP = 11772kW$

$D_o = 3.5m$

$D_b = 1.75m$

$\alpha = 35°$

$\eta_h = 88\%$

$\eta_o = 84\%$

$V_{w2} = 0 \, m/s$

→ A Kaplan turbine runner is to be designed to develop 9100kW. The net available head is 5.6m. If the speed ratio = 2.09, flow ratio = 0.68, overall efficiency 86% and the diameter of the boss is 1/3 the diameter of the runner. Find the diameter of the runner, its speed and specific speed.

Soln: $S.P = 9100 kW$

$H = 5.6 m$

$\phi = 2.09$

$C_v = 0.68$

$\eta_o = 86/100$

$D_b = \frac{1}{3} D_o$

$u_1 = 2.09\sqrt{2\times9.81\times5.6}$

$= 21.95 m/s$

$V_{f1} = 0.68\sqrt{2\times9.81\times5.6}$

$= 7.12 m/s$

$$\eta_o = \frac{S.P}{\frac{\cancel{g}\times\rho\times Q\times H}{\cancel{1000}}}$$

$$\frac{86}{100} = \frac{9100}{\frac{1000\times9.81\times Q\times5.6}{1000}}$$

$Q = 192.5 m^3/s$

$$Q = \frac{\pi}{4}\left[D_o^2 - D_b^2\right]\times V_{f1}$$

$$192.5 = \frac{\pi}{4}\left[D_o^2 - \frac{1}{9}D_o^2\right]\times7.12$$

$D_o = 6.21 m$

$$u_1 = \frac{\pi D_o N}{60}$$

$N = 67.5 rpm$

$$\text{Specific speed} = \frac{N\sqrt{P}}{H^{5/4}} = \frac{67.5\times\sqrt{9100}}{5.6^{5/4}}$$

$= 746$

$$\eta_o = \frac{S.P}{W.P}$$

$$\frac{84}{100} = \frac{11{,}772}{1000 \times 9.81 \times Q \times 20}$$

$$Q = 71.428\ m^3/s$$

$$Q = \frac{\pi}{4}\left(D_o^2 - D_b^2\right) \times V_{f_1}$$

$$71.428 = \frac{\pi}{4}\left[3.5^2 - 1.75^2\right] \times V_{f_1}$$

$$= \frac{\pi}{4}\left(12.25 - 3.0625\right) V_{f_1}$$

$$= 7.216\ V_{f_1}$$

$$V_{f_1} = \frac{71.428}{7.216} = 9.9\ m/s$$

$$\tan\alpha = \frac{V_f}{V_{w_1}}$$

$$\tan 35^\circ = \frac{9.9}{V_{w_1}}$$

$$V_{w_1} = 14.14\ m/s$$

$$\eta_H = \frac{V_{w_1} u_1}{gH}$$

$$0.88 = \frac{14.14 \times u_1}{9.81 \times 20}$$

$$u_1 = 12.21\ m/s$$

(ii) Runner vane angles at inlet and outlet

$$\tan\theta = \frac{V_{f_1}}{V_{w_1} - u_1} = \frac{9.9}{14.14 - 12.21}$$

$$\theta = \tan^{-1}(5.13)$$

$$= 78^\circ 58'$$

$$u_1 = u_2 = 12.21\ m/s, \quad V_{f_1} = V_{f_2} = 9.9\ m/s$$

$$\tan\phi = \frac{V_{f_2}}{u_2} = \frac{9.9}{12.21} = 0.811$$

$$\phi = 39^\circ 2'$$

$$u_1 = u_2 = \frac{\pi D_o N}{60}$$

$$12.21 = \frac{\pi \times 3.5 \times N}{60}$$

$$N = 66.63\ r.p.m$$

A Kaplan turbine runner is to be designed to develop 7357.5 kW shaft power the net available head is 5.5 m Take $\phi = 2.09$ and $C_v = 0.68$, $\eta_o = 60\%$. $D_b = \frac{1}{3} \times D_o$ Find the diameter of the runner and its speed.

Soln

$S.P = 7357.5\ kW$

$H = 5.5m$

$\phi = 2.09$

$C_v = 0.68\ m/s$

$\eta_o = 60\%$

$D_b = \frac{1}{3} \times D_o$

$$u_1 = \phi\sqrt{2gH}$$
$$= 2.09\sqrt{2 \times 9.81 \times 5.5}$$
$$= \underline{21.71\ m/s}$$

$$V_{f_1} = C_v\sqrt{2gH}$$
$$= 0.68 \times \sqrt{2 \times 9.81 \times 5.5}$$
$$= 7.064\ m/s$$

$$\eta_o = \frac{S.P}{W.P}$$

$$0.60 = \frac{7357.5}{\frac{1000 \times Q \times 9.81 \times 5.5}{1000}}$$

$$Q = 227.27\ m^3/s$$

$$Q = \frac{\pi}{4}\left[D_o^2 - D_b^2\right] \times V_{f_1}$$
$$= \frac{\pi}{4}\left[D_o^2 - \left(\frac{D_o}{3}\right)^2\right] \times 7.064$$

$$227.27 = 4.9316\ D_o^2$$

$$D_o = 6.788\ m$$

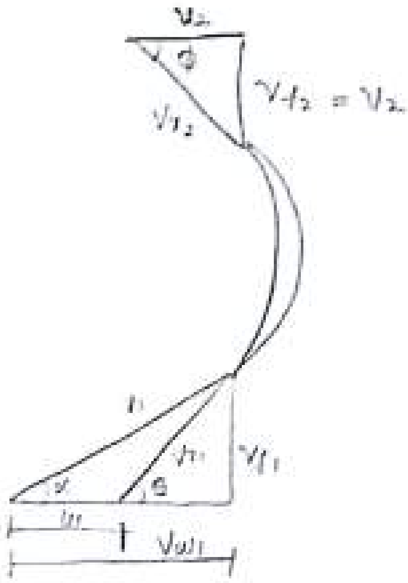
$$D_b = \frac{1}{3} \times 6.785$$
$$= \underline{2.262m}$$

$$u_1 = \frac{\pi D_o N}{60}$$

$$N = \frac{60 \times u_1}{\pi D_o}$$

$$N = 61.08\ rpm$$

(2) The hub diameter of a Kaplan turbine, working under a head of 12m, is 0.35 times the diameter of the runner. If the turbine is running at 100 rpm, if the vane angle of the extreme edge of the runner at outlet is 15°, and flow ratio 0.6, find (i) diameter of the runner (ii) Diameter of the boss (iii) Discharge through the runner. The velocity of whirl is zero at outlet.



Soln

$H = 12m$

$D_b = 0.35 D_o$

$N = 100 rpm$

$\phi = 15°$

$C_v = 0.6$

$D_o = ?$

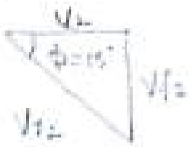
$D_b = ?$

$Q = ?$

$$V_{f_1} = C_v\sqrt{2gH}$$
$$= 0.6\sqrt{2\times9.81\times12}$$
$$= 9.2\ m/s$$

$$V_{f_1} = V_{f_2} = 9.2\ m/s$$

Outlet Δ^{le}



$$\tan 15° = \frac{V_{f_2}}{u_2}$$
$$u_2 = \frac{9.2}{\tan 15°}$$
$$u_2 = 34.33\ m/s$$
$$u_1 = u_2 = 34.33\ m/s$$

$$u_1 = \frac{\pi D_o N}{60}$$

$$\frac{34.33\times60}{\pi\times100} = D_o$$

$$D_o = 6.55\ m$$

$$D_b = 0.35\times6.55$$
$$= 2.3\ m$$

$$Q = \frac{\pi}{4}(D_o^2 - D_b^2)\times V_f = \frac{\pi}{4}\left[6.55^2 - 2.3^2\right]\times9.2$$
$$= \underline{271.77\ m^3/s}$$

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Draft tube :

A draft tube is a pipe or passage of gradually increasing cross sectional area which connects the runner exit to the tail race. It may be made of cast or plate steel or concrete. It must be airtight and under all conditions of operation its lower end must be submerged below the level of water in the tail race.

It serves the following purposes :

1) It permits a negative or suction head to be established at the runner and thereby increasing the net head on the turbine.

2) It converts a large proportion of the kinetic energy $\left(\frac{V_2^2}{2g}\right)$ rejected at the outlet of the turbine to useful pressure energy.

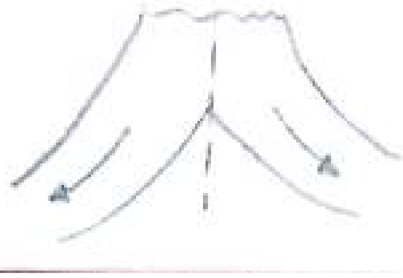
Types of Draft Tube :

1) Conical draft tube : It takes the form of the frustum of a cone, with central angle less than 8°. It has an efficiency of 90% and generally employed for low specific speed and vertical shaft turbines.

2) Simple elbow tube : It finds the application where the length of the shaft has to be minimum so as to cut short the volume of excavation. Efficiency is of 60%.

3) Moody spreading draft tube.

This type of draft tube is provided with a central core of conical shape which reduces whirling action of discharging water. Its efficiency is about 85%.



b) Elbow draft tube with circular inlet and rectangular outlet

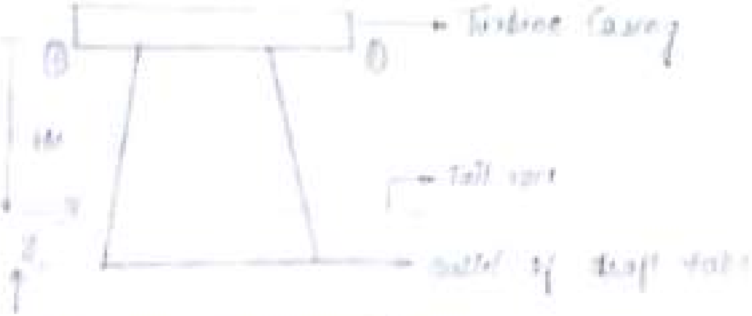
In this type, the change from circular section in the vertical leg to rectangular section in the horizontal leg takes place in the bend. Efficiency is about 85%.

Draft tube Theory:

Consider a draft tube as shown

H_s - vertical height of draft tube above tail race

y - distance of bottom of draft tube from tail race



Applying Bernoulli's eqn

$$\frac{p_1}{\rho g} + \frac{V_1^2}{2g} + H_s + y = \frac{p_2}{\rho g} + \frac{V_2^2}{2g} + h_f$$

$$\frac{p_1}{\rho g} = \frac{p_2}{\rho g} - H_s - \left[\frac{V_1^2}{2g} - \frac{V_2^2}{2g}\right] - h_f$$

Efficiency of a Draft tube

$$\eta_d = \frac{\left[\frac{V_1^2}{2g} - \frac{V_2^2}{2g}\right] - h_f}{\left[\frac{V_1^2}{2g}\right]}$$

(7)

→ A conical draft tube having inlet and outlet diameter 1m and 1.5m discharges water at outlet with a velocity of 2.5 m/s the total length of the draft tube is 6m and 1.2 m of the length of draft tube is immersed in water If the atmospheric pressure head is 10.3 m of water and loss of head due to friction in the draft tube is equal to 0.2 times the velocity head at outlet of the tube. find (i) Pressure head at inlet

(ii) Efficiency of the draft tube

given

$d_1 = 1m$

$d_2 = 1.5 m$

$V_2 = 2.5 m/s$

$H_s + y = 6m$

$y = 1.2m$

$H_s = 6 - 1.2 = 4.80m$

$\frac{p_a}{\rho g} = 10.3 m$

$h_f = 0.2 \times \frac{V_2^2}{2g}$

$Q_2 = A_2 \times V_2$

$= \frac{\pi (1.5)^2}{4} \times 2.5$

$= 4.4178 m^3/s$

$Q_1 = A_1 \times V_1$

$4.4178 = \frac{\pi (1)^2}{4} \times V_1$

$V_1 = 5.625 m/s$

$$\frac{p_1}{\rho g} = \frac{p_a}{\rho g} - H_s - \left[\frac{V_1^2}{2g} - \frac{V_2^2}{2g}\right] - h_f$$

$$= 10.3 - 4.8 - \left[\frac{5.625^2}{2 \times 9.81} - \frac{2.5^2}{(2 \times 9.81)}\right] - 0.2 \frac{2.5^2}{2 \times 9.81}$$

$$= \underline{4.27 m}$$

$$\eta_d = \frac{\left[\frac{V_1^2}{2g} - \frac{V_2^2}{2g}\right] - h_f}{\left[\frac{V_1^2}{2g}\right]}$$

$$= \underline{76.3 \%}$$

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Centrifugal Pumps

Introduction

The hydraulic machines which convert the mechanical energy into hydraulic energy are called Pumps

The hydraulic energy is in the form of pressure energy

If the mechanical energy is converted into pressure energy by means of centrifugal force acting on the fluid, the hydraulic machine is called Centrifugal pump

In this type of pump the liquid is subjected to whirling motion by the rotating impeller which is made of a number of backward curved vanes. The liquid enters this impeller at its centre or the eye and gets discharged into the casing enclosing the outer edge of the impeller

The rise in the pressure head at any point of the impeller is proportional to the square of the tangential velocity of the liquid at that point. i.e $\frac{u^2}{2g}$

Hence at the outlet of the impeller where the radius is more, the pressure head will be more and the liquid will be discharged at the outlet with a high pressure head

Due to high pressure head, the liquid can be lifted to a higher level.

Advantages of Centrifugal pump.

→ Its initial cost is low

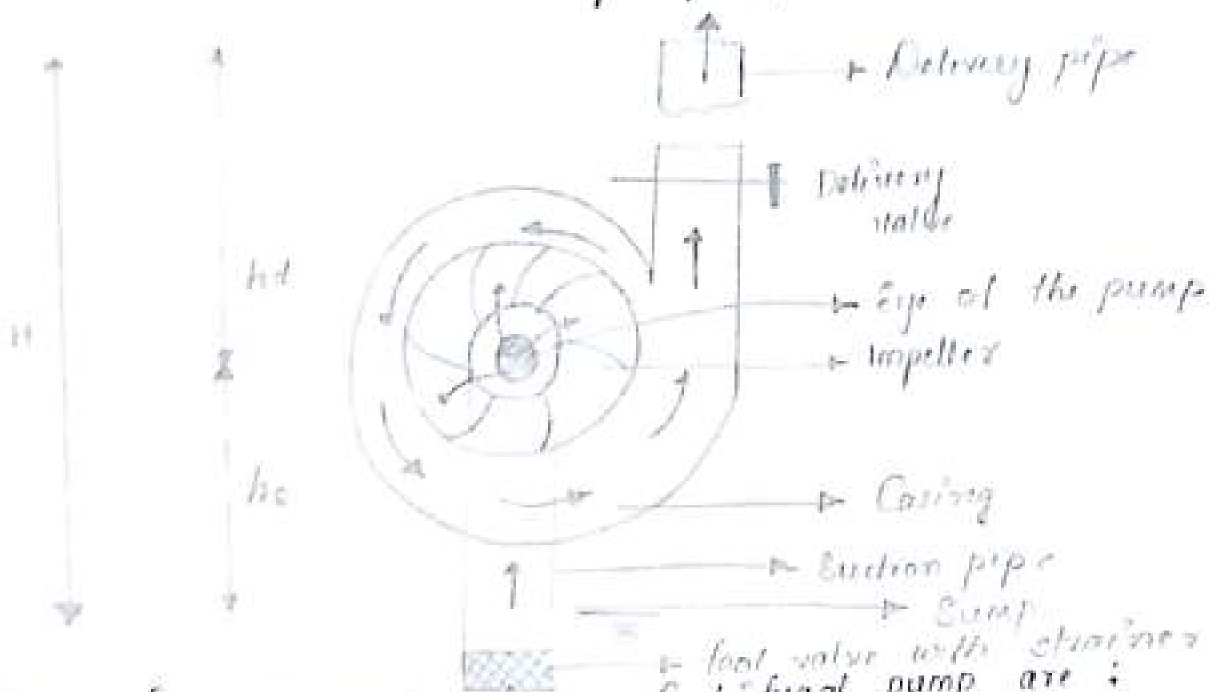
→ Efficiency is high

→ Discharge is uniform and continuous

→ Installation and maintenance is easy

→ It can run at high speeds, without the risk of seperation of flow

Main Parts of a Centrifugal pump



The main components of a Centrifugal pump are :
1> Impeller <2> Casing <3> Suction pipe <4> Foot valve with strainer <5> Delivery pipe <6> Delivery valve

Impeller : it is the rotating component of the pump It is made up of a series of curved vanes. The impeller is mounted on the shaft connecting an electric motor

Casing : It is an air tight chamber surrounding the impeller the shape of the casing is designed in such a way that the kinetic energy of the impeller is gradually changed to potential energy. This is achieved by gradually increasing the area of c/s in the direction of flow

Suction Pipe : It is the pipe connecting the pump to the sump from where the liquid has to be lifted up

Foot valve with strainer : The foot valve is a non-return valve which permits the flow of the liquid from the sump towards the pump It only opens in the upward direction The strainer is a mesh surrounding the valve, it prevents the entry of debris and silt into the pump

Delivery pipe : It is a pipe connected to the pump to the overhead tank.

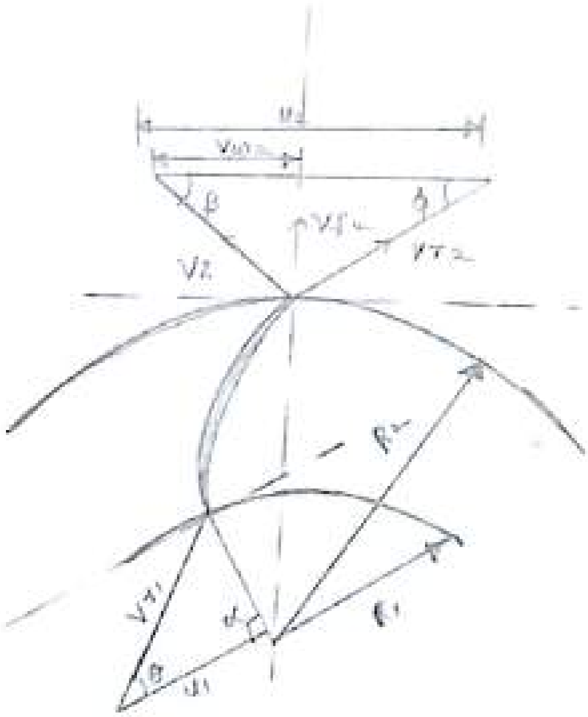
Delivery valve : It is a valve which can regulate the flow of liquid from the pump.

PRIMING OF A CENTRIFUGAL PUMP

Priming of a centrifugal pump is defined as the operation in which the suction pipe, casing of the pump and a portion of the delivery pipe upto the delivery valve is completely filled up from outside source with the liquid to be raised by the pump before starting the pump. Thus the air from these parts of the pump is removed by the impeller and these parts are filled with liquid to be pumped.

WORK DONE BY THE CENTRIFUGAL PUMP (OR IMPELER) ON WATER

The velocity triangles for the impeller at inlet and outlet is as shown in the figure



$V_{w1} = 0$

$\alpha = 90°$

The water enters the impeller radially at inlet for best efficiency of the pump, which means the absolute velocity of water at inlet makes an angle of 90° with the direction of motion of the impeller at inlet. Hence angle $\alpha = 90°$ and $V_{w_1} = 0$.

Let N = Speed of the impeller in rpm.

D_1 = Diameter of impeller at inlet.

u_1 = Tangential velocity of impeller at inlet

$= \frac{\pi D_1 N}{60}$

D_2 = Diameter of impeller at outlet

u_2 = tangential velocity of impeller at outlet.

$= \frac{\pi D_2 N}{60}$

V_1 = absolute velocity of water at inlet

V_{r_1} = relative velocity of water at inlet

α = angle made by absolute velocity (V_1) at inlet with the direction of motion of vane

θ = angle made by relative velocity (V_{r_1}) at inlet with the direction of motion of vane.

V_{r_2}, β and ϕ are the corresponding values at outlet.

The work done by the water on the runner per sec per unit weight of the water striking per sec is given by the equation as

$$= \frac{1}{g}\left[V_{w_1} u_1 - V_{w_2} u_2\right]$$

$\therefore$ Work done by the impeller on the water per sec per unit weight of water striking per sec

$= -$ [work done in case of turbine]

$$= \frac{1}{g} V_{w_2} u_2$$

Work done by impeller on water per second

$$= \frac{W}{g} V_{w2} u_2 \qquad W = \text{weight of water} = \rho \times g \times Q$$

Q = Area $\times$ velocity of flow

$= \pi D_1 B_1 \times V_{f1}$

$= \pi D_2 B_2 \times V_{f2}$

B_1 and B_2 are width of impeller @ inlet and outlet and V_{f1} and V_{f2} are velocities of flow @ inlet and outlet.

HEADS OF A CENTRIFUGAL PUMP.

1) Suction head (h_s): It is the vertical height of the centre line of the centrifugal pump above the water surface in the tank or pump from which water is to be lifted. this height is called suction lift and denoted as 'h_s'

2) Delivery head (h_d): The vertical distance between the centre line of the pump and the water surface in the tank to which water is delivered is known as delivery head and denoted as 'h_d'.

3) Static head (H_s): The sum of suction head and delivery head is known as static head and denoted as 'H_s'

$$H_s = h_s + h_d$$

4) Manometric head (H_m): The manometric head is defined as the head against which a centrifugal pump has to work. It is denoted as 'H_m'

a) $H_m = \frac{V_{w2} u_2}{g}$ – loss of head.

$= \frac{V_{w2} u_2}{g}$ if loss is zero.

b) $H_m = \left[\frac{p_o}{\rho g} + \frac{V_o^2}{2g} + Z_o\right] - \left[\frac{p_i}{\rho g} + \frac{V_i^2}{2g} + Z_i\right]$

$\frac{p_o}{\rho g}, \frac{p_i}{\rho g}$ = pressure head at outlet of the pump and inlet of the pump

$\frac{V_o^2}{2g}, \frac{V_i^2}{2g}$ = velocity head at outlet and inlet of the pump

Z_o, Z_i = datum head at outlet and inlet

c) $H_m = h_s + h_d + h_{fs} + h_{fd} + \frac{V_d^2}{2g}$

h_s - suction head

h_d = delivery head

h_{fs} = frictional head loss in suction pipe

h_{fd} = " " " in delivery pipe

V_d = Velocity of water in delivery pipe

Efficiencies of a Centrifugal pump

In centrifugal pump, power is transmitted from shaft of the electric motor to the shaft of the pump and then to the impeller and then to the water. The power keeps on decreasing.

i) Manometric efficiency: The ratio of power given to water at outlet of the pump to the power available at the impeller is known as Manometric efficiency

Power at outlet = $\frac{WH_m}{1000}$ kW

Power at impeller = $\frac{\text{Work done by impeller per sec}}{1000}$ kW

$= \frac{W}{g} \times \frac{V_{w_2} \times u_2}{1000}$

$$\eta_{man} = \frac{g \times H_m}{V_{w_2} \times u_2}$$

* Mechanical efficiency (η_m): The ratio of the power available at the impeller to the power at the shaft of the pump is known as Mechanical efficiency.

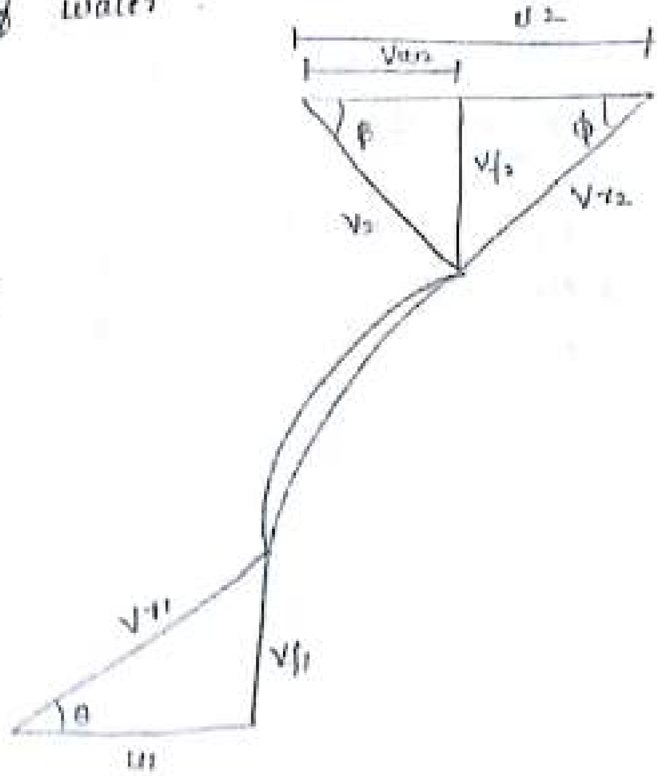
$$\eta_m = \frac{\frac{W}{g}\left[V_{w_2} u_2\right]/1000}{\text{Shaft power}}$$

* Overall efficiency (η_o): It is the ratio of power output of the pump to the power input to the pump

$$\eta_o = \frac{\frac{WH_m}{1000}}{\text{Shaft power}}$$

$$\eta_o = \eta_{man} \times \eta_m$$

P 26 The internal and external diameters of the impeller of a centrifugal pump are 200mm and 400 mm respectively. The pump is running at 1200 rpm. The vane angles of the impeller at inlet and outlet are 20° and 30° respectively. The water enters the impeller radially and velocity of flow is constant. Determine the work done by the impeller per unit weight of water.



Soln:
$D_1 = 200mm = 200\times10^{-3}$ m

$D_2 = 400mm = 400\times10^{-3}$ m.

$N = 1200$ rpm

Vane angle at inlet $= \theta = 20°$

Vane angle at outlet $= \phi = 30°$

$V_{f1} = V_{f2}$

$\alpha = 90°$, $V_{w1} = 0$

$$u_1 = \frac{\pi D_1 N}{60}$$

$$= \frac{\pi \times 200\times10^{-3} \times 1200}{60}$$

$$= 12.56 \text{ m/s}$$

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$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 400 \times 10^{-3} \times 1200}{60}$

$= 25.13 \, m/s$

from inlet velocity Δ^{le}

$\tan\theta = \frac{V_{f1}}{u_1}$

$\tan 20° = \frac{V_{f1}}{12.56}$

$V_{f1} = 4.57$

$V_{f2} = 4.57$

$\tan\phi = \frac{V_{f2}}{u_2 - V_{w2}}$

$\tan 30° (25.13 - V_{w2}) = 4.57$

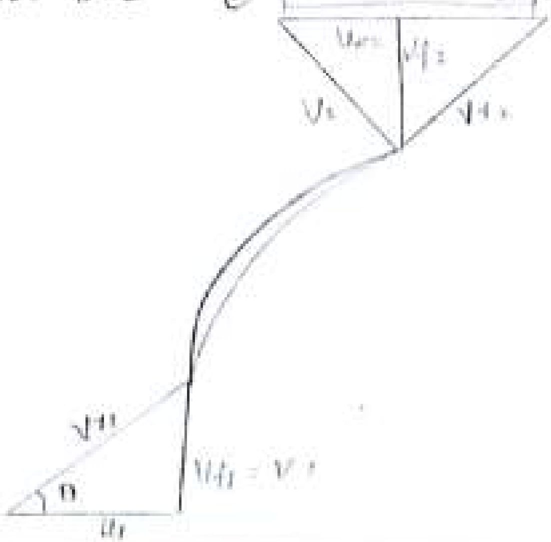
$V_{w2} = 17.215 \, m/s$

Work done by impeller per unit weight of water

$= \frac{1}{g} V_{w2} u_2 = \frac{1}{9.81} \times 17.215 \times 25.13$

$= 44.1 \, Nm/N$

2) A centrifugal pump having outer diameter equal to 2 times the inner diameter and running at 1000 rpm against a total head of 40m The velocity of flow through the impeller is constant and equal to 2.5 m/s The vanes are set back at an angle of 40° at outlet If the outer diameter of the impeller is 500mm Determine ① Vane angle ② Work done by impeller on water per sec ③ Manometric efficiency.



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$N = 1000\,rpm$

$H_m = $ [illegible]

$V_{f_1} = V_{f_2} = 2.5\ m/s$

$\phi = 40^\circ$

$D_2 = 500mm = 0.5\ m$

$D_1 = \frac{D_2}{2} = \frac{0.5}{2} = 0.25m$

$B_2 = 0.05m$

$$u_1 = \frac{\pi D_1 N}{60} = \frac{\pi \times 0.25 \times 1000}{60} = 13.09\ m/s$$

$$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.5 \times 1000}{60} = 26.18\ m/s$$

$$Q = \pi D_2 B_2 \times V_{f_2} = \pi \times 0.5 \times 0.05 \times 2.5$$

$$= 0.1963\ m^3/s$$

$$\tan\theta = \frac{V_{f_1}}{u_1} = \frac{2.5}{13.09} = 0.191$$

$$\theta = \tan^{-1}(0.191) = 10^\circ 48'$$

Work done by impeller on water per second

$$= \frac{\rho \times g \times Q}{g} \times V_{w_2} \times u_2$$

$$= \frac{1000 \times 9.81 \times 0.1963 \times V_{w_2} \times 26.18}{9.81}$$

$$\tan\phi = \frac{V_{f_1}}{u_2 - V_{w_2}} = \frac{2.5}{26.18 - V_w}$$

$$V_{w_2} = 23.2\ m/s$$

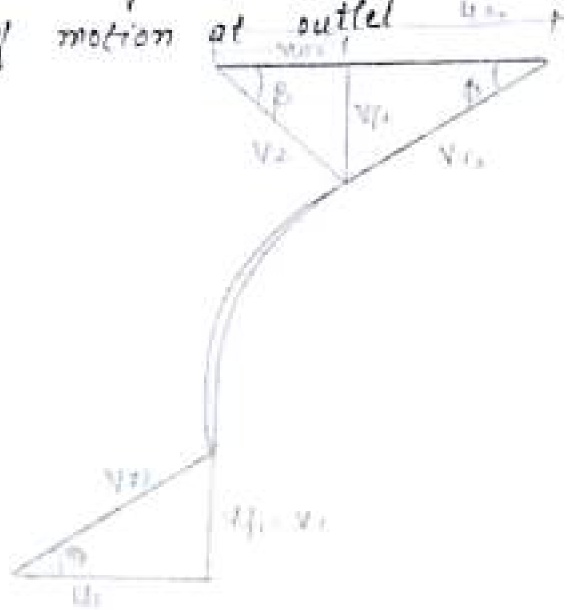
$\therefore$

$$= \frac{\rho \times g \times Q \times V_{w_2} \times u_2}{9.81}$$

$$= \frac{1000 \times 9.81 \times 0.1963 \times 23.2 \times 26.18}{9.81}$$

$$= 119227.9\ Nm/s$$

3) The outer diameter of an impeller of a centrifugal pump is 400mm and outlet width 50mm The pump is running at 800 rpm and is working against a Total head of 15m. The vanes angle at outlet is 40° and manometric efficiency is 75%. Determine the (i) velocity of flow at inlet (ii) velocity of water leaving the vane (iii) angle made by absolute velocity at outlet with the direction of motion at outlet (iv) Discharge



$D_2 = 0.4m$

$B_2 = 0.05m$

$N = 800 rpm$

$Hm = 15m$

$\eta_{man} = 75\%$

$\phi = 40°$

$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 0.4 \times 800}{60}$

$= 16.75 m/s$

$\eta_{man} = \frac{gHm}{V_{w_2} u_2}$

$0.75 = \frac{9.81 \times 15}{V_{w_2} \times 16.75}$

$V_{w_2} = 11.71 m/s$

$\tan\phi = \frac{V_{f_2}}{u_2 - V_{w_2}} = \frac{V_{f_2}}{5.04}$

$V_{f_2} = 5.04 \times \tan\phi$

$= 4.23 m/s$

$V_2 = \sqrt{V_{f_2}^2 + V_{w}^2}$

$= 12.45 m/s$

$\tan\beta = \frac{V_{f_2}}{V_{w_2}} = \frac{4.23}{11.71} = 0.36$

$\beta = 19°48'$

$Q = \pi D_2 B_2 \times V_{f_2}$

$= \pi \times 0.4 \times 0.05 \times 4.23$

$= 0.265 m^3/s$

Find the power required to drive a centrifugal pump which delivers $0.04\,m^3/s$ of water to a height of 20m through a 15cm dia pipe and 100m long. The overall efficiency of the pump is 70% and co-efficient of friction $f = 0.15$ in the formula $h_f = \frac{4fLV^2}{2gd}$.

$Q = 0.04\,m^3/s$

$H = 20m$

$d = 15\times10^{-2}\,m$

$L = 100m$

$\eta_o = 70\%$

$f = 0.15$

$$\text{Velocity} = \frac{\text{Discharge}}{\text{area of pipe}} = \frac{0.04}{\frac{\pi(15\times10^{-2})^2}{4}} = 2.26\,m/s$$

$$h_{fs} + h_{fd} = \frac{4fLV^2}{2gd} = \frac{4\times0.15\times100\times2.26^2}{2\times9.81\times15\times10^{-2}}$$

$$= 10.41\,m$$

$$H_m = h_s + h_d + h_{fs} + h_{fd} + \frac{V_d^2}{2g}$$

$$= 20 + 10.41 + \frac{2.26^2}{2\times9.81}$$

$$= 30.67\,m$$

$$\eta_o = \frac{WH_m}{1000/S.P}$$

$$= \frac{\rho g\,Q\times H_m}{1000\times S.P}$$

$$S.P = 17.19\,KW$$

A centrifugal pump is running at 1000rpm. The outlet vane angle of the impeller is 45° and velocity of flow at outlet is 2.5m/s. The discharge through the pump is 200lps when the pump is working against a total head of 20m. If the manometric efficiency of the pump is 80%, Determine ① dia of impeller ② width of impeller at outlet.

$N = 1000\ rpm$

$\theta = 45^\circ$

$V_{f2} = 2.5\ m/s$

$Q = 200\ l/s$

$H_m = 20\ m$

$\eta_{mano} = 80\%$

$$\tan\theta = \frac{V_{f2}}{u_2 - V_{w2}}$$

$$u_2 - V_{w2} = \frac{V_{f2}}{\tan 45^\circ}$$

$$\boxed{V_{w2} = u_2 - 2.5}$$

$$\eta_{mano} = \frac{gH_m}{V_{w2}\, u_2}$$

$$0.80 = \frac{9.81 \times 20}{V_{w2}\, u_2}$$

$$V_{w2}\, u_2 = 245.25$$

$$V_{w2} = \frac{245.25}{u_2}$$

$$V_{w2} = u_2 - 2.5$$

$$\frac{245.25}{u_2} = u_2 - 2.5$$

$$u_2^2 - 2.5u_2 - 245.25$$

$$u_2 = 16.95\ m/s$$

$$u_2 = \frac{\pi D_2 N}{60}$$

$$D_2 = 0.32\ m$$

$$Q = \pi B_2 D_2 V_{f2}$$

$$0.2 = \pi \times B_2 \times 0.32 \times 2.5$$

$$B_2 = 0.078\ m$$

Minimum Speed for starting a Centrifugal Pump.

If the pressure rise in the impeller is more or equal to the manometric head, then the centrifugal pump will start delivering water.

When impeller is rotating, the water in contact with the impeller also rotates. This is a case of forced vortex.

In case of forced vortex, head due to pressure rise in the impeller is given by

$$= \frac{\omega^2 r_2^2}{2g} - \frac{\omega^2 r_1^2}{2g}$$

ωr_2 = Tangential velocity of impeller @ outlet $= u_2$

ωr_1 = " " " " @ inlet $= u_1$

head due to pressure rise in the impeller $= \frac{u_2^2}{2g} - \frac{u_1^2}{2g}$

For the centrifugal pump to deliver water

$$\frac{u_2^2}{2g} - \frac{u_1^2}{2g} \geq H_m.$$

$$\eta_{man} = \frac{gH_m}{V_{w_2} u_2}$$

$$H_m = \frac{V_{w_2} \times u_2}{g} \times \eta_{man}.$$

$$\frac{u_2^2}{2g} - \frac{u_1^2}{2g} = \frac{V_{w_2} \times u_2}{g} \times \eta_{man}$$

$$\frac{1}{2g}\left[\frac{\pi D_2 N}{60}\right]^2 - \frac{1}{2g}\left[\frac{\pi D_1 N}{60}\right]^2 = \eta_{max} \times \frac{V_{w_2}}{g}\left[\frac{\pi D_2 N}{60}\right]$$

$$\frac{\pi D_2^2 N}{120} - \frac{\pi D_1^2 N}{120} = \eta_{max} \times V_{w_2} \times D_2$$

$$\frac{\pi N}{120}\left[D_2^2 - D_1^2\right] = \eta_{max} \times V_{w_2} \times D_2$$

$$N = \frac{120 \times \eta_{max} \times V_{w_2} \times D_2}{\pi\left[D_2^2 - D_1^2\right]}$$

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The diameters of an impeller of a centrifugal pump at inlet and outlet are 30 cm and 60 cm respectively. Determine the min starting speed of the pump if it works against a head of 30 m.

$D_1 = 30\times10^{-2}\,m$

$D_2 = 60\times10^{-2}\,m$

$H_m = 30\,m$

For minimum speed

$$\frac{u_2^2}{2g} - \frac{u_1^2}{2g} = H_m$$

$$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi(60\times10^{-2})N}{60}$$

$$u_1 = \frac{\pi D_1 N}{60} = \frac{\pi(30\times10^{-2})N}{60}$$

$$\frac{\pi^2 D_2^2 N^2}{2\times g\times 60^2} - \frac{\pi^2 D_1^2 N^2}{2\times g\times 60^2} = 30$$

$$\frac{1}{2g}\left[0.0314 N\right]^2 - \frac{1}{2g}\left[0.0157 N\right]^2 = 30$$

$$N^2 = 795297.7$$

$$\boxed{N = 892\ rpm}$$

A centrifugal pump with 1.2m dia runs at 200rpm and pumps 1880 lps, the average lift being 6m. The angle with which the vanes make an exit with the tangent to the impeller is 26° and the radial velocity of flow is 2.5 m/s. Determine the manometric efficiency and the least speed to start pumping against a head of 6m, the inner diameter of the impeller being 0.6m

$D_2 = 1.2m$

$N = 200 rpm$

$Q = 1880 lps = 1880 \times 10^{-3} m^3/s$

$\phi = 26°$

$V_{f2} = 2.5 m/s$

$D_1 = 0.6m$

$H_m = 6m$

$$\eta_{max} = \frac{gH_m}{V_{w2} \times u_2}$$

$$u_2 = \frac{\pi D_2 N}{60} = \frac{\pi \times 1.2 \times 200}{60}$$

$$= 12.56 m/s$$

$$\tan\phi = \frac{V_{f2}}{u_2 - V_{w2}}$$

$$u_2 - V_{w2} = \frac{V_{f2}}{\tan 26°}$$

$$V_{w2} = 7.43 m/s$$

$$\eta_{max} = \frac{9.81 \times 6}{7.43 \times 12.56}$$

$$= 63\%$$

Least speed to start the pump

$$\frac{u_2^2}{2g} - \frac{u_1^2}{2g} = H_m$$

$$N = \frac{120 \times \eta_{max} \times V_{w2} \times D_2}{\pi [D_2^2 - D_1^2]}$$

$$= \frac{120 \times 63/100 \times 7.43 \times 1.2}{\pi [1.2^2 - 0.6^2]}$$

$$= 200 rpm$$

Priming of a Centrifugal Pump.

Priming of a centrifugal pump is defined as the operation in which the suction pipe, casing of the pump and a portion of the delivery pipe upto the delivery valve is completely filled up from outside source with the liquid to be raised by the pump before starting the pump.
Thus the air from these parts of the pump is removed and these parts are filled with the liquid to be pumped.

When the pump is running in air, the head generated is in terms of metre of air. If the pump is primed with water, the head generated is same as metre of water. Since the density of air is very low, the generated head of air in terms of equivalent metre of water head is negligible and hence the water may not be sucked from the pump. To avoid this difficulty, priming is necessary.

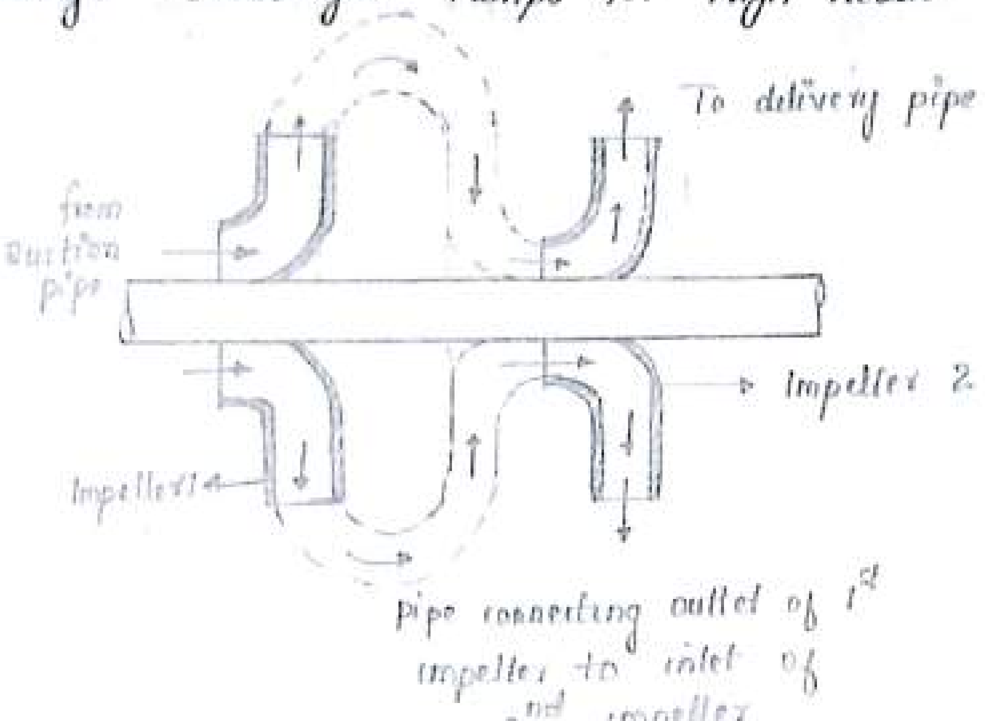
Multistage Centrifugal Pump.

If a centrifugal pump consists of two or more impellers, the pump is called a Multistage Centrifugal pump. The impellers may be mounted on the same shaft or on different shafts. A multistage pump is having the following functions.

(i) To produce high head

(ii) To discharge a large quantity.

Multistage Centrifugal Pumps for High heads



[Two stage pump in Series]

For developing a high head, a number of impellers are mounted in series

The water from the suction pipe enters the first impeller at inlet and is discharged at outlet with increased pressure. The water with increased pressure from the outlet of the first impeller is taken to the inlet of the second impeller with the help of a connecting pipe At the outlet of 2nd impeller. the pressure of water will be more than the pressure of water at the outlet of the first impeller

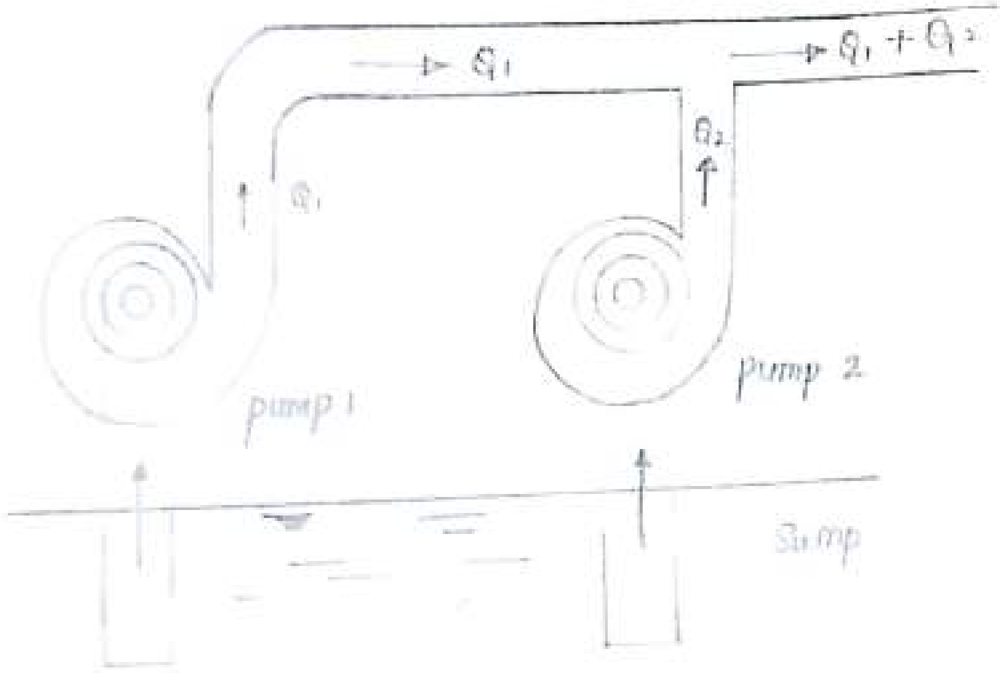
Therefore more impellers mounted. the pressure at the outlet will be increased

Total head = $n \times H_m$

n = no of impelleres

H_m = Head developed by each impeller

Multistage Centrifugal Pumps for high Discharge.



Pumps in Parallel

For obtaining high discharge, the pumps should be connected in parallel as shown. Each of the pumps lifts the water from a common pump and discharges water to a common pipe to which the delivery pipe of each pump is connected Each of the pump will work against the same head

Total Discharge $= n \times Q$

n = no of identical pumps

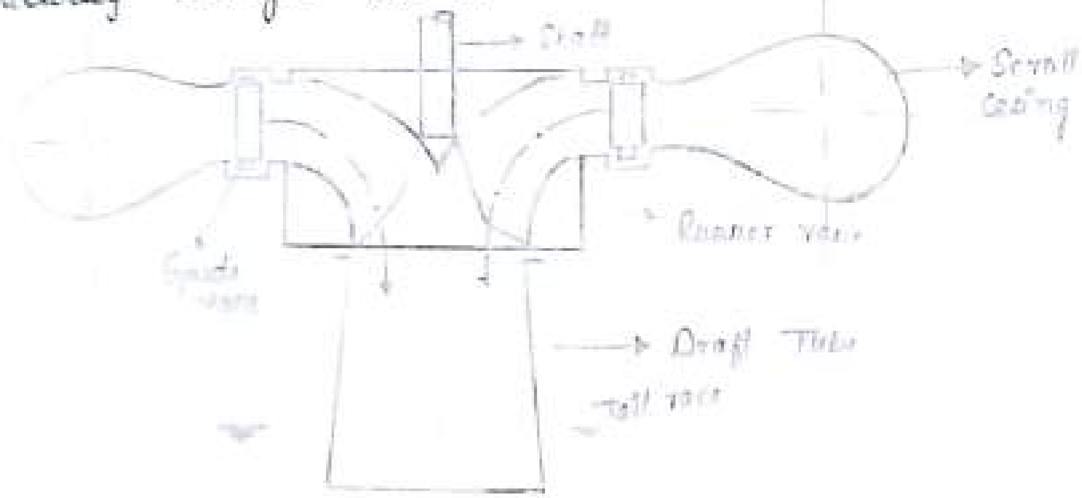
Q - discharge from each pump

FRANCIS TURBINE

If the water flows from outwards to inwards radially, then the turbine is known as inward radial flow turbine. Francis Turbine is an inward flow reaction turbine which was designed and developed by the American engineer James B Francis in the early stages of its development, the Francis turbine was a radial flow runner type. But the modern Francis turbine is a mixed flow type in which the water enters the runner radially at its outer periphery and leaves it in the axial direction.

Work done by the turbine runner

velocity triangles are drawn as shown below.



Sectional Arrangement of Francis Turbine

General expression for work done from the momentum equation is given by,

$$\text{work done} = \rho Q \left[V_{w_1} u_1 \pm V_{w_2} u_2 \right]$$

In Francis turbine, the velocity of whirl at outlet i.e. ($V_{w_2} = 0$) will be zero.

Hence the work done by water on the runner per sec will be

$$= \rho Q [V_{w1} u_1]$$

And work done per sec per unit weight of water striking 1 sec.

$$= \frac{1}{g} [V_{w1} u_1]$$

Hydraulic efficiency, $\eta_h = \frac{V_{w1} u_1}{gH}$.

Working Proportions of Francis Runner.

i) The ratio of width of the wheel to its diameter is given as $n = \frac{B}{D}$ and its value ranges from 0.10 to 0.45.

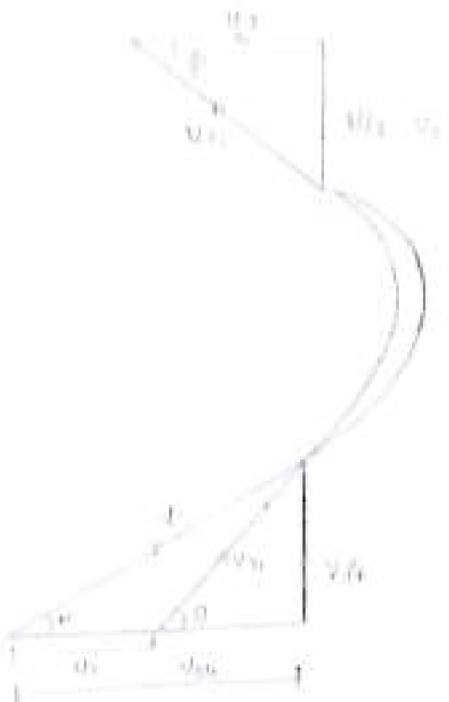
ii) flow ratio $= \frac{V_{f1}}{\sqrt{2gH}}$ and varies from 0.15 to 0.30.

iii) The speed ratio $= \frac{u_1}{\sqrt{2gH}}$ and varies from 0.6 to 0.9.

The following data is given for a Francis turbine. Net head $H = 60m$, speed $N = 700rpm$, Shaft power $= 294.3kW$, $\eta_0 = 84\%$, $\eta_h = 93\%$, flow ratio $= 0.20$, breadth ratio $n = 0.1$, Outer dia of the runner $= 2 \times$ inner diameter of runner. The thickness of vanes occupy 5% of circumferential area of the runner. velocity of flow is constant at inlet and outlet and discharge is radial at outlet. Determine

(i) Guide blade angle (ii) Runner vane angles at inlet and outlet

(iii) Diameter of runner at inlet and outlet (iv) Width of wheel at inlet



Soln: $H = 60m$

$N = 700rpm$

$S.P = 294.3kW$

$\eta_0 = 84\%$

$\eta_H = 93\%$

flow ratio $= 0.20$

breadth ratio, $n = 0.1$

$D_1 = 2 \times D_2$

$n = 0.1$

$\boxed{\text{Flow Area} = 0.95 \times \pi \times B \times D_1}$

$V_{f_1} = V_{f_2} = V_2$

since discharge is radial $V_{w_2} = 0$

$$\boxed{\eta_0 = \frac{S.P}{W.P}}$$

$$\boxed{W.P = \frac{\rho \times Q \times g \times H}{1000}}$$

$$\frac{84}{100} = \frac{294.3}{W.P}$$

$W.P = 350.35\,kW$

$1000 \times 350.35 = (1000 \times 9.81 \times Q \times 60)$

$Q = 0.594\ m^3/s$

$$\boxed{V_{f_1} = 0.20\sqrt{2gH}}$$

$V_{f_1} = 0.20 \times \sqrt{2 \times 9.81 \times 60}$

$V_{f_1} = 6.862\ m/s$

$$\boxed{n = \frac{B_1}{D_1}}$$

$B_1 = 0.1 \times D_1$

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$$\boxed{\text{Discharge} = \text{Area of flow} \times \text{velocity of flow}}.$$

$$0.5952 = 0.95 \times \pi \times D_1 \times B_1 \times V_{f_1}$$

$$0.5952 = 0.95 \times \pi \times D_1 \times 0.1 \times D_1 \times 6.862.$$

$$D_1^2 = 0.2906$$

$$D_1 = 0.54\, m.$$

$$B_1 = 0.1 D_1$$

$$= 0.1 \times 0.54$$

$$B_1 = 0.054\, mm.$$

Tangential speed of the runner at inlet

$$\boxed{u_1 = \frac{\pi D_1 N}{60}}$$

$$= \frac{\pi \times 0.54 \times 700}{60}$$

$$u_1 = 19.8\, m/s.$$

Hydraulic efficiency

$$\boxed{\eta_H = \frac{V_{w_1} u_1}{gH}}$$

$$\frac{93}{100} = \frac{V_{w_1} \times 19.8}{9.81 \times 60}$$

$$V_{w_1} = 27.66\, m/s.$$

Guide blade angle α

$$\boxed{\tan\alpha = \frac{V_{f_1}}{V_{w_1}}} = \frac{6.862}{27.66} = 0.248.$$

$$\alpha = \tan^{-1}(0.248)$$

$$\alpha = 13^\circ 55.7'.$$

Runner vane angles at inlet and outlet. (θ and ϕ)

$$\boxed{\tan\theta = \frac{V_{f_1}}{V_{w_1} - u_1}} = \frac{6.862}{27.66 - 19.79} = 0.872.$$

$$\theta = \tan^{-1}(0.872) = 41^\circ 5'.$$

$$\boxed{\tan\phi = \frac{V_{f_2}}{u_2}}$$

$$\tan\phi = \frac{V_{f_1}}{u_2}.$$

$$\boxed{u_2 = \frac{\pi D_2 N}{60}}$$

$$D_2 = \frac{D_1}{2}$$

$$u_2 = \pi \times \frac{0.54}{2} \times \frac{700}{600}$$

$$u_2 = \underline{9.896} \text{ m/s}$$

$$\tan\phi = \frac{6.862}{9.896}$$

$$= 0.6934$$

$$\phi = \tan^{-1}(0.6934)$$

$$\phi = 34^\circ 44'$$

An inward flow reaction turbine works under a head of 30m and discharge of 10 m³/sec the speed of the runner is 300 rpm the inlet tip of runner vane, the peripheral velocity of wheel is $0.9\sqrt{2gH}$ and the radial velocity of flow is $0.3\sqrt{2gH}$, where H is the head on the turbine H the overall efficiency and the hydraulic efficiency of the turbine are 80% and 90% respectively. Determine (a) the power developed in KW

(b) Diameter and width of runner at inlet

(c) guide blade at inlet

(d) diameter of runner at outlet

Discharge at outlet is radial.

Soln: $H = 30m$.

$Q = 10 m^3/s$.

$N = 300 rpm$.

$u_1 = 0.9\sqrt{2gH}$.

$V_{f1} = 0.3\sqrt{2gH}$

$\eta_o = 80\%$

$\eta_h = 90\%$.

$V_{w2} = 0$.

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$u_1 = 0.9\sqrt{2\times 9.81\times 30}$

$= 21.83\ m/s$.

$V_{f1} = 0.3\sqrt{2\times 9.81\times 30}$

$= 7.27\ m/s$.

Overall efficiency.

$$\eta_o = \frac{\text{Shaft Power}}{\text{Water Power}}$$

$$\text{Shaft Power} = \eta_o \times S \times g \times Q \times H$$

$$= \frac{80}{100} \times 1000 \times 9.81 \times 10 \times 30$$

$$= 2354.4\ KW.$$

$$u_1 = \frac{\pi D_1 N}{60}.$$

$$\frac{21.83 \times 60}{\pi \times 300} = D_1$$

$$D_1 = 1.4m.$$

$$Q = \pi D_1 B_1 \times V_{f1}$$

$$10 = \pi \times 1.4 \times B_1 \times 7.27$$

$$B_1 = 0.315m.$$

Hydraulic efficiency.

$$\eta_H = \frac{V_{w1} u_1}{gH}.$$

$$\frac{90}{100} \times \frac{9.81 \times 30}{21.83} = V_{w1}$$

$$V_{w1} = 12.13\ m/s.$$

$$\tan\alpha = \frac{V_{f1}}{V_{w1}}$$

$$\tan\alpha = \frac{7.27}{12.13}$$

$$\alpha = \tan^{-1}(0.6)$$

$$\alpha = 30^\circ.$$

$$\tan\theta = \frac{V_{f_1}}{V_{w_1} - u_1}$$

$$= \frac{7.27}{12.13 - 21.83}$$

$$= -0.74 \Rightarrow \theta = \tan^{-1}(0.74) = -36^\circ 51'$$

$$\theta = 143^\circ 9'$$

An inward flow reaction turbine is required to develop 394kW at 220rpm. The effective head on the turbine is 20m. Determine the inside and outside diameters. inlet and exit angles for the vanes and the guide blade angles. Assume inlet diameter to be twice the outlet diameter, hydraulic efficiency as 80% and constant radial velocity of flow 3.5m/s. Assume ratio of width of wheel at inlet to its diameter as 0.1 and that 5% of the area of flow in the runner is blocked by the blades. Assume radial discharge.

Soln

$S \cdot P = 294 \times 10^3 W$

$N = 220 rpm$

$H = 20 m$

D_1 & D_2 = ?

$\eta_h = 82\%$

$V_{f_1} = 3.5 m/s$

$D_1 = 2D_2$

$\frac{B}{D} = 0.1$

$$\eta_H = \frac{V_{w_1} u_1}{gH}$$

$$9.81 \times 20 \times \frac{82}{100} = V_{w_1} u_1$$

$$V_{w_1} u_1 = 160.885$$

$$\eta_o = \frac{S \cdot P}{W \cdot P}$$

$$\frac{80}{100} = \frac{294 \times 10^3}{\rho \times Q \times g \times H}$$

$$\frac{80}{100} = \frac{294 \times 10^3}{1000 \times Q \times 9.81 \times 20}$$

$$Q = 1.875 m^3/s$$

$$Q = 0.95 \times \pi \times B_1 \times D_1 \times V_{f_1}$$

$$1.875 = 0.95 \times \pi \times 0.1 \times D_1^2 \times 3.5$$

$$D_1^2 = 1.795$$

$$D_1 = 1.34 m$$

$$B_1 = 0.1 \times 1.34$$

$$B_1 = 0.134 m$$

$$D_2 = \frac{D_1}{2}$$

$$= \frac{1.34}{2}$$

$$D_2 = 0.67 m$$

$$u_1 = \frac{\pi D_1 N}{60}$$

$$= \frac{\pi \times 1.34 \times 220}{60}$$

$$u_1 = 15.42 m/s$$

$$u_2 = \frac{\pi D_2 N}{60}$$

$$= \pi \times \frac{0.67 \times 220}{60}$$

$$= 7.71 m/s$$

from equation (1)

$$V_{w1} u_1 = 160.385$$

$$V_{w1} = \frac{160.385}{15.42}$$

$$V_{w1} = 10.43 \text{ m/s}$$

Guide blade angle α

$$\tan\alpha = \frac{V_{f1}}{V_{w1}} = \frac{3.5}{10.43}$$

$$= 0.335$$

$$\alpha = \tan^{-1}(0.335)$$

$$\alpha = 18^\circ 33'$$

$$\tan\phi = \frac{V_{f2}}{u_2}$$

$$= \frac{3.5}{7.71}$$

$$= 0.454$$

$$\phi = \tan^{-1}(0.454)$$

$$\phi = 24^\circ 24'$$

$$\tan\theta = \frac{V_{f1}}{V_{w1} - u_1}$$

$$= \frac{3.5}{10.43 - 15.42}$$

$$\theta = -35^\circ 2'$$

$$\theta = 180^\circ - 35^\circ 2'$$

$$\theta = 144^\circ 58'$$